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ABSTRACT

Expectation Shocks and Learning as Drivers of the Business Cycle

Psychological factors, market sentiments, and shifts in beliefs are believed by many to play a nontrivial role in inducing and amplifying economic fluctuations. Yet, these forces are rarely considered in macroeconomic models. This paper provides an attempt to evaluate the empirical role of expectational shocks on business cycle fluctuations. The paper relaxes the conventional assumption of rational expectations to exploit observed data on survey and market expectations in the estimation of a benchmark New Keynesian model. The observed expectations are modeled as formed from a near-rational expectation formation mechanism, which assumes that economic agents use a linear perceived law of motion for economic variables that has the same structural form as the model solution under rational expectations and that they need to learn model coefficients over time. In addition to the typical structural demand, supply, and policy disturbances, the model incorporates expectation shocks, which affect the formation of expectations by the private sector. Both the best-fitting learning process and the expectations shocks are identified from the expectations data and from the interaction between expectations and realized data. The expectations shocks capture waves of optimism and pessimism that lead agents to form forecasts that deviate from those implied by their learning model and by the state of the economy. The empirical results uncover a crucial role for these novel expectations shocks as a major driving force of the U.S. business cycle. Expectation shocks regarding future real activity are the main source of economic fluctuations, since they can account for roughly half of business cycle fluctuations.

JEL Classification: E31, E32, E52 and E58
Keywords: behavioral explanations of the business cycle, constant-gain learning, DSGE estimation with survey expectations, expectation formation, expectation shocks and waves of optimism and pessimism

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1. Introduction

Macroeconomists have been seeking for a long time to identify the causes of economic fluctuations. Empirical work has not reached definitive conclusions, but many researchers would agree that a variety of technology shocks, demand shock, monetary and fiscal policy shocks, in varying percentages, are responsible for the bulk of the business cycle.

In the past, however, economists also emphasized the importance of less conventional explanations for cyclical fluctuations. Psychological variables, in particular, were thought to play a crucial role in causing and amplifying business cycles. Keynes (1936), for example, attributed cycles to the action of "animal spirits", while Pigou (1927) discussed how business people's "errors of undue optimism or undue pessimism in their business forecasts" created fluctuations in industrial activity.

Similar explanations, however, are rarely at the center of the current generation of macroeconomic models. Their omission likely arises from the pervasive difficulty in measuring expectational or psychological shifts from observed realizations of macroeconomic variables.[1]

New Keynesian models, which are often used to characterize the interaction between macroeconomic variables and monetary policy, share this limitation, as they are similarly based on the idea that fluctuations are driven by exogenous structural shocks to technology, households' preferences, firms' mark-ups, and to policies.

Yet, disturbances related to the formation of expectations, waves of optimism and pessimism, periods of generalized exuberance or gloom, which may be unrelated to fundamentals, may contribute in non-trivial part to economic fluctuations and, in such case, they should be taken into consideration in the formulation of monetary policy.

The main contribution of this paper is to propose a way to re-introduce these psychological elements in a monetary business cycle model, with the objective of investigating their contribution to economic activity. More specifically, the paper provides an attempt to evaluate the empirical importance of expectational shocks – which may be interpreted as exogenous changes in the private sector's degree of optimism or pessimism – as a source of aggregate economic fluctuations. These shocks affect the formation of expectations and cause changes in expectations that are unrelated to economic fundamentals, by making private economic agents more optimistic, or pessimistic, about the future state of the economy than it would be justified if they simply formed expectations from their perceived model of the economy and with beliefs derived from current and historical data.

The paper, therefore, relaxes the conventional assumption of rational expectations. To capture expectational swings, the paper exploits time series data on observed expectations (mostly derived from the Survey of Professional Forecasters), along with real-time data, to estimate a baseline

[1]Partial exceptions are the literatures on sunspots and news shocks, which will be discussed later in this section.

New Keynesian model, which departs from the previous literature by including a potential role for psychological forces. The observed expectations are assumed to be formed from a near-rational expectations formation mechanism. Economic agents adopt a perceived model of the economy that has a similar structural form to the rational expectations solution of the system. Agents, however, do not know the reduced-form coefficients of the system, but they can observe historical data on variables as output, inflation, and interest rates (in some cases, also unobserved disturbances as natural rate, cost-push, or policy shocks will be assumed to be in the agents' information set). Therefore, they exploit historical series to attempt to learn the reduced-form coefficients over time through constant-gain learning. They form expectations in each period from their perceived model, using the most recently updated parameter estimates and the data available in real-time.

The model with learning is found to be a good approximation of the expectations formation from the survey. The model, however, allows economic agents to depart from the numeric forecasts implied by the learning model. Private sector agents in some periods may be overly optimistic – by forecasting a higher future output or lower inflation rate, for example, than implied by their learning model – or overly pessimistic. These waves of over-optimism and over-pessimism, which are exogenous to the state of the economy, are defined as the expectation shocks in the model. Different specific expectation shocks are allowed to affect the formation of output, inflation, or monetary policy expectations.

Survey data on expectations are exploited to extract both the best-fitting evolution of agents' beliefs and the expectation shocks over the sample. Both the agents' learning process and the properties of expectational shifts are thus not imposed a priori, but they are estimated from time series data on expectations, and from the dynamic interaction between expectations and realized variables within the structural model. The learning and expectation formation coefficients are jointly estimated along with the remaining structural parameters.

Preview of the Results. The empirical results reveal a large role for expectational shocks. These "optimism" and "pessimism" shocks, in particular related to future expectations about economic activity, are found to be a major source of business cycle fluctuations. Expectation shocks explain roughly half of business cycle movements, while the structural demand, supply, and policy shocks that have been typically considered in the literature explain the remaining half.

Fundamental demand shocks also have a large effect on output in the short run, but in a model that incorporates observed expectations and learning, they are far less persistent than found in previous literature. The adjustment of the economy after demand shocks is much faster than commonly implied by monetary DSGE models. The output gap response peaks only after few months after the shock and quickly vanishes to zero. Expectation shocks, on the other hand, cause a substantially more persistent adjustment. The effect on output is larger, delayed, and more long-lived than the corresponding effect provoked by structural demand shocks.

Fluctuations in inflation are also mostly driven by expectational shocks related to future real activity and future inflationary pressures. The impact of cost-push shocks on inflation is sizable on impact, but it dissipates rather quickly.

The most important shock for the business cycle is, therefore, an expectational shock (an optimism/pessimism shift regarding forecasts of future real activity) that moves output and inflation in the same direction and thus looks as a demand shock and may be misinterpreted as one in a typical estimation.

Turning to expectations data, the observed expectations do respond to developments in the state of the economy. But often their main determinant is an exogenous expectational component, which is unrelated to fundamentals and which explains about 60% of fluctuations in the expectation variables.

Related Literature. The paper is related to various strands of the literature. First, and more directly, the paper contributes to the literature on the role of learning and expectations in macroeconomics. While a large portion of the literature on adaptive learning is interested in studying the convergence properties of systems with learning to the rational expectations equilibrium (e.g., Evans and Honkapohja, 1999, 2001), a more recent set of papers aims to analyze the empirical role of learning in the economy during the transition phase using time series data. Milani (2006, 2007a), for example, estimates a monetary business cycle model to show that learning can successfully capture the persistence of macroeconomic variables, thus rendering some of the commonly employed "mechanical" sources of persistence, such as habit formation in consumption and inflation indexation, possibly redundant. Other papers by Adam (2004) and Orphanides and Williams (2005a,b) also demonstrate the role of learning in inducing persistence in macroeconomic variables. This paper is particularly related to the work by Eusepi and Preston (2008), who use a real-business cycle model with learning by private agents to study the impact of learning dynamics on the business cycle. They show that endogenous changes in expectations, due to learning, can play a role in amplifying the transmission of technology shocks, since they allow the model to match the volatility of output with a standard deviation of the technology shock that remains 20-30% percent smaller than the one required in the same standard RBC specification, but under rational expectations (an earlier work with similar objective is Williams, 2003). This paper considers both endogenous changes in the learning process and exogenous expectation shocks to show that they play a large role in a monetary model, which is estimated exploited observed expectation data.

The learning specification used in this paper is closely related to the one used in Bullard, Evans, and Honkapohja (2008, 2010). Bullard et al. add a term in the agents' perceived law of motion that they define as "judgment", i.e. the addition of factors in forecasting that are extraneous to their model. Their work concludes that, under such expectation mechanism, the economy can converge to near-rational exuberance equilibria, which are characterized by higher volatility than the corresponding rational expectations equilibrium. This paper, instead, considers a similar learning specification and uses survey data to extract exogenous expectational shocks. The paper then shows that these expectational shocks are a major force in driving economic fluctuations.

A methodological contribution of the paper consists in the use of data on expectations to estimate a general equilibrium model with learning by economic agents and in providing an approach to identify and evaluate psychological factors as contributors to economic fluctuations. Milani (2007a) estimates models with learning, but using only realized macroeconomic variables, and not data on forecasts. Other papers estimated models with learning (e.g., Milani, 2006, 2007b, 2008, 2009,a,b, Slobodyan and Wouters, 2007) in a similar fashion. This paper is, instead, related to recent work by Ormeno (2009), who also exploits observed expectations to estimate models under learning. Ormeno considers only inflation forecasts, while this paper incorporates data on all the forecasted variables that enter the New Keynesian model. The focus in Ormeno's work is different and he doesn't introduce expectation shocks, which are a major focus here. Carboni and Ellison (2009) use available Greenbook forecasts to study and explain Federal Reserve policy during the Great Inflation. Early work by Orphanides and Williams (2005b) also uses inflation survey to inform the choice of the constant gain coefficient that better approximates the private sector's learning process.

Information about observed expectations has been also recently used by Del Negro and Eusepi (2009) to judge whether DSGE models under rational expectations fit inflation expectations from surveys. They conclude that rational expectations models do only a poor job in explaining existing inflation forecasts.

Moreover, within the extensive literature on the main sources of the economic fluctuations, the paper is more closely related to the studies that propose "behavioral" theories for the business cycle.

A substantial literature in macroeconomics highlights the importance of self-fulfilling fluctuations driven by sunspots.[2] Sunspots cause shifts between multiple equilibria and induce fluctuations in economic activity. As an implication of the importance of expectational shocks, the model presented in this paper can generate self-fulfilling fluctuations without requiring the existence of

[2]E.g., Azariadis (1981), Cass and Shell (1983), Benhabib and Farmer (1994, 1999), Farmer and Woodford (1997), and Woodford (1986, 1990, 1991).

sunspots and multiple equilibria. Self-fulfilling fluctuations can arise in the model without the need to assume increasing returns to scale, for example, or, in a New Keynesian model, without requiring a monetary policy rule that violates the Taylor principle (as in Lubik and Schorfheide, 2004, or Castelnuovo and Surico, 2010, for example). Positive shifts in market sentiment may lead to the formation of expectations that overestimate the output forecast that would be justified by fundamentals and by the their updated learning model. These variations in optimism and pessimism are to some extent self-fulfilling, since errors of optimism have a positive effect on the actual level of output. Besides leading to potential self-fulfilling fluctuations, sentiment shifts may also amplify the impact of fundamental shocks: the interaction between fundamental and expectational shocks may lead to fluctuations that reinforce each other, generating large business cycle fluctuations.

A related body of research that emphasizes expectation-driven business cycles is the "news" shocks literature. The current work shares with papers on "news" shocks[3] its focus on how expectations matter for the business cycle. The approach used in this paper, however, clearly differs. While the papers in the news literature, in fact, usually emphasize the importance of news about future technology changes, the expectational shocks introduced here do not represent information about future fundamentals, but they are thought as capturing shifts in market sentiment that is unrelated to current fundamentals and to near-rationally formed expectations about future fundamentals.

Other papers present behavioral explanations of the business cycle. De Grauwe (2009) studies animal spirits in an otherwise standard New Keynesian model that arise from agents' use of simple biased rules for forecasting output and inflation. He shows that heuristic rules can give rise to endogenous cycles. Angeletos and La'o (2009) develop a theoretical model, which assumes heterogeneous information about real shocks hitting the economy: they show that in the model business cycle fluctuations are driven by what they define as "noise", i.e. correlated errors in expectations about technology shocks. This paper models a very different mechanism, but it shares with the previous papers the focus in identifying disturbances related to expectations as an important source of fluctuations. Moreover, the paper clearly departs from the mentioned studies, as it is mainly empirical in focus. The focus on structural estimation is, instead, shared by Blanchard, L'Huillier, and Lorenzoni (2009), who explore the role of "noise" shocks, defined as temporary errors in economic agents' estimates regarding future fundamentals. They use maximum likelihood estimation of a structural model and find that noise shocks play an important role in creating short-run fluctuations in economic activity.

[3]E.g., Beaudry and Portier (2006), Jaimovich and Rebelo (2009), Christiano, Ilut, Motto, and Rostagno (2007), Lorenzoni (2009), Schmitt-Grohé and Uribe (2008), Milani and Treadwell (2009).

The paper is also more broadly connected to the literature on the estimation of DSGE models. These studies usually consider a large variety of structural disturbances (to technology, preferences, mark-ups, and so forth): in Smets and Wouters (2007), for example, the evidence seems to suggest that mark-up shocks in the labor market are the most important source of fluctuations. This paper highlights, instead, the role of a usually omitted source of fluctuations. In this line of work, expectations are pinned down by structure of the economy: expectations are part of the transmission mechanism, but they do not constitute an independent source of fluctuations. This paper, instead, departs from the conventional view and uncovers a possibly large role for expectational shocks that are orthogonal to the structure of the economy.

2. Model

I assume a baseline New Keynesian model as a description of the behavior of macroeconomic variables as output, inflation, and interest rates (e.g., Woodford, 2003, Galí, 2008):

$$y_t = \widehat{E}_{t-1}\left[y_{t+1} - \sigma\left(i_t - \pi_{t+1} - r_t^n\right)\right] \quad (2.1)$$

$$\pi_t = \widehat{E}_{t-1}\left[\beta\pi_{t+1} + \kappa y_t + u_t\right] \quad (2.2)$$

$$i_t = \rho i_{t-1} + (1-\rho)[r_t^n + \chi_\pi \pi_{t-1} + \chi_y y_{t-1}] + \varepsilon_t. \quad (2.3)$$

This simple framework has been widely used in the monetary policy literature and it forms the building block of the medium and large-scale models that are now often used to characterize the U.S. economy (e.g., Christiano, Eichenbaum, and Evans, 2005, Smets and Wouters, 2007).

Equation (2.1) is the loglinearized Euler equation that arises from the households' optimal choice of consumption. The current output gap, denoted by y_t, depends on expectations about future output gap in $t+1$, and on the deviation of the ex-ante real interest rate (given by the difference between the expected nominal interest rate i_t and the expected inflation rate in $t+1$, denoted by π_{t+1}) from the natural rate r_t^n. The coefficient σ denotes the elasticity of intertemporal substitution of private expenditures. The natural rate r_t^n acts as a disturbance in the IS equation, and it moves in response to aggregate taste, technology, and government spending shocks in the economy.

Equation (2.2) represents a New Keynesian Phillips curve. The current inflation rate π_t depends on the expected inflation rate in $t+1$ and on output gap in period t. Inflation is also affected by the exogenous cost-push shock u_t, which can be endogenously derived by assuming time-varying elasticity of substitution among different varieties of goods, for example. The coefficient β denotes the households' discount factor, while κ is a composite parameter, which denotes the slope of the Phillips curve and which is a function of coefficients indicating the degree of price rigidity in the economy, the Frisch elasticity of labor supply, and the elasticity of substitution among differentiated goods, among others.

Equation (2.3) denotes a Taylor rule with partial adjustment, which serves as an approximation of monetary policy decisions in the economy. The monetary authority is assumed to set the policy instrument, a short-term nominal interest rate, in response to movements in inflation and the output gap. The reaction coefficients are denoted by χ_π and χ_y, while ρ accounts for the observed inertia of interest rate decisions. The assumed rule is operational in the sense of McCallum (1999), as it requires the central bank to have knowledge only of the recent values of the variables in $t-1$, rather than the contemporaneous values in t. Deviations from the systematic monetary policy rule are captured by the monetary policy shock ε_t. While the natural rate and the cost-push shocks evolve as AR(1) processes as $r_t^n = \rho_r r_{t-1}^n + \sigma_r \nu_t^r$ and $u_t = \rho_u u_{t-1} + \sigma_u \nu_t^u$, where $\nu_t^r, \nu_t^u \sim N(0,1)$, the monetary policy shock is assumed to be i.i.d. Normal with mean 0 and standard deviation σ_ε.

The model departs in two ways from the benchmark New Keynesian framework.

The assumption of rational expectations is relaxed. In the model, $\widehat{E}_t$ will, in fact, correspond to observed survey and market expectations, for which I will exploit actual data in the estimation, rather than rational, model-consistent, expectations. I will assume that the observed expectations are formed by agents from a near-rational expectations formation mechanism, the details of which will be provided in the next section.

Moreover, the model assumes that expectations are predetermined as in Giannoni and Woodford (2003): economic agents dispose only of information up to $t-1$ when forming expectations about variables in t and $t+1$ and when solving their maximization problems ($\widehat{E}_{t-1}$ hence replaces $\widehat{E}_t$ in the model). This assumption is made here for empirical reasons, i.e. to match the timing in the Survey of Professional Forecasters and the information set (only up to $t-1$) that is available to the survey forecasters when forecasting period t and $t+1$ variables (the assumption that agents dispose only of $t-1$ information when forming expectations is, however, common in the adaptive learning literature, e.g. Evans and Honkapohja, 2001, as it permits to avoid simultaneity issues). One implication of the assumption of predetermined expectations is that forecasts about future monetary policies also explicitly enter the model and affect consumers' decisions.

One caveat should be usually noticed about the model's microfoundations under learning. I have assumed a model that is characterized by the same loglinearized equations that are obtained under rational expectations: only expectations of variables up to $t+1$ matter for the dynamics of current macroeconomic variables. Under subjective expectations and learning, however, Preston (2005, 2006, 2008) shows that long-horizon expectations may also enter the model. Honkapohja, Mitra, and Evans, (2003) discuss the conditions under which Preston's approach simplifies to yield the model in this paper (more importantly, the requirement that agents need to recognize that the market clearing condition $y_t = c_t$, where c_t indicates consumption, holds at all times). This paper's choice of using the so-called Euler-equation approach, rather than the infinite-horizon approach, follows the majority of papers in the adaptive learning literature; moreover, the choice is motivated by the availability of expectations data from the survey, which includes one-quarter and two-quarter-ahead expectations for a long sample, but lacks a variety of long-horizon expectations data. Extending the empirical analysis to an infinite-horizon framework, however, is important for future research.

The choice of a benchmark New Keynesian model, instead, is meant to make the paper comparable to a vast body of empirical literature, which used the same framework, and to render the results on the role of expectation shocks as transparent as possible.

2.1. **Expectations Formation in Real-Time.** Typically, DSGE models assume that expectations are formed according to the rational expectations hypothesis. Under this assumption, expectational errors may be solved out as a function of the structural disturbances and eliminated from the system. Expectations are, therefore, unequivocally pinned down by the structure of the economy. Economic agents are assumed to have knowledge about the parameters of the economy, the correct model and its solution, the distributions of the shocks, and so forth. While this remains the standard approach in empirical macroeconomics, this paper attempts to look at the empirical evidence on the sources of economic fluctuations through a different lens.

The paper, therefore, abandons the conventional assumption of rational expectations and exploits, instead, survey and market data on expectations, which will be treated as observable variables in the estimation. I still assume that economic agents form expectations from a model of expectations formation, which aims to explain the observed expectations' data. The expectation formation mechanism consists of a rather small deviation from model-consistent rational expectations. Economic agents are assumed to form expectations according to a perceived law of motion, which has similar structural form to the minimum state variable solution of the model under rational expectations (i.e., the same observable regressors that appear in the MSV solution under rational expectations also appear in the agents' perceived model). The Perceived Law of Motion (PLM) is, therefore, given by

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = a_t + b_t \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + \epsilon_t, \tag{2.4}$$

which resembles a VAR(1) in the model's endogenous variables. In contrast to the rational expectations case, however, agents are assumed to lack knowledge about the reduced-form parameters of the economy (for example, they lack knowledge about some aggregate parameters, such as the Calvo pricing parameter, or as other households' preference parameters, and, as a consequence, they cannot infer the reduced-form coefficients in the model solution). They are also assumed not to be able to observe the realizations of the shocks. Therefore, they use only observable variables as output, inflation, and interest rates, and not unobserved disturbances, in their regressions. This is seen as the most empirically realistic case, but, later in the paper, I will re-estimate the model under the alternative case in which agents are endowed with knowledge about the disturbances as well, so that the PLM corresponds to the minimum state variable solution under rational expectations: the results are unchanged. In this framework, agents in the model share the same knowledge that an econometrician estimating the model would have in real time.

Although the constant in the model solution under rational expectations will be equal to zero, economic agents are not endowed with this information and, therefore, they learn about the intercepts as well. In this way, the learning specification can permit to capture agents' misperceptions about the steady-state levels of inflation and interest rates and about the level of potential output or its trend.

Economic agents try to infer the reduced-form parameters in (2.4) using the following constant-gain learning algorithm, through the updating rules

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + \overline{\mathbf{g}} R_t^{-1} X_t \left(Y_t - X_t' \widehat{\phi}_{t-1} \right) \tag{2.5}$$

$$R_t = R_{t-1} + \overline{\mathbf{g}} \left(X_t X_t' - R_{t-1} \right) \tag{2.6}$$

where $Y_t \equiv [y_t, \pi_t, i_t]'$ is a vector of endogenous variables, $X_t \equiv \{1, Y_{t-1}\}$ is a matrix of regressors, and $\widehat{\phi}_t = (a_t', vec(b_t'))'$ collects the reduced-form coefficients. The first expression (2.5) describes the updating of agents' beliefs, while the second expression (2.6) illustrates the updating of the precision matrix R_t corresponding to the stacked regressors X_t. These expressions correspond to a recursive formulation of the weighted least squares estimator. Agents learn about the relevant coefficients by revising their previous period estimates in the direction of the most recent forecast errors. A crucial coefficient under learning is the constant gain coefficient $\overline{\mathbf{g}}$, which governs the rate at which agents discount past information when forming expectations about future macroeconomic variables. Different values of the gain coefficient permit to approximate substantially different evolutions of the learning process.

Expectations about future variables in t and $t+1$ are formed each period using the PLM (2.4) along with the most recently updated coefficients $\widehat{\phi}_t$ from (2.5), as:[4]

$$\widehat{E}_{t-1} \begin{pmatrix} y_t \\ \pi_t \\ i_t \end{pmatrix} = a_t + b_t \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{pmatrix} + \begin{pmatrix} e_t^{y_0} \\ 0 \\ e_t^i \end{pmatrix}, \tag{2.7}$$

[4]One may wonder whether there is evidence that expectations are formed in a way that is consistent with the PLM and learning model previously described. In current parallel work, I'm investigating whether the assumed learning model provides a good approximation of the expectation formation process of individual forecasters from the SPF. Branch and Evans (2006) show that constant-gain learning is indeed a good approximation of aggregate inflation and output growth forecasts from the SPF.

and

$$\widehat{E}_{t-1}\left(\begin{array}{c} y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \end{array}\right) = a_t + b_t \widehat{E}_{t-1}\left(\begin{array}{c} y_t \\ \pi_t \\ i_t \end{array}\right) + \left(\begin{array}{c} e_t^{y_1} \\ e_t^{\pi} \\ 0 \end{array}\right). \tag{2.8}$$

In the empirical section, I will use time series data on one-period-ahead and two-period-ahead expectations $\widehat{E}_{t-1}y_t$, $\widehat{E}_{t-1}y_{t+1}$, $\widehat{E}_{t-1}\pi_{t+1}$, and $\widehat{E}_{t-1}i_t$, as observable variables. The variables $e_t^{y_0}$, $e_t^{y_1}$, e_t^{π}, and e_t^i are the expectations shocks (there are no expectations shocks related to $\widehat{E}_{t-1}\pi_t$ and $\widehat{E}_{t-1}i_{t+1}$, as these expectations do not enter the model). The shock $e_t^{y_0}$ indicates the expectational shock that refers to the output forecast between $t-1$ and t, while $e_t^{y_1}$ indicates the slightly longer-horizon shock related to the output forecast between t and $t+1$. The two-period-ahead forecast, therefore, includes the effect of two disturbances, the same disturbance as the one-period-ahead forecast and a second longer-horizon disturbance.

The expectations shocks are allowed to be persistent: e_t^{π} and e_t^i follow the AR(1) processes

$$e_t^{\pi} = (1-\rho_e^{\pi})\bar{\rho}_e^{\pi} + \rho_e^{\pi} e_{t-1}^{\pi} + \sigma_e^{\pi}\tilde{e}_t^{\pi} \tag{2.9}$$

$$e_t^{i} = (1-\rho_e^{i})\bar{\rho}_e^{i} + \rho_e^{i} e_{t-1}^{i} + \sigma_e^{i}\tilde{e}_t^{i}, \tag{2.10}$$

where σ_e^{π} and σ_e^i denote the standard deviations of the expectational innovations, where $\tilde{e}_t^{\pi}$, $\tilde{e}_t^i$ $\sim N(0,1)$. The expectations shocks related to future output, instead, are allowed to be dependent on each other. They evolve as a VAR(1):

$$\left[\begin{array}{c} e_t^{y_1} \\ e_t^{y_0} \end{array}\right] = \left[\begin{array}{c} (1-\rho_e^{y_1})\bar{\rho}_e^{y_1} \\ (1-\rho_e^{y_0})\bar{\rho}_e^{y_0} \end{array}\right] + \left[\begin{array}{cc} \rho_e^{y_1} & \rho_{y_1,y_0} \\ \rho_{y_0,y_1} & \rho_e^{y_0} \end{array}\right]\left[\begin{array}{c} e_{t-1}^{y_1} \\ e_{t-1}^{y_0} \end{array}\right] + \left[\begin{array}{cc} \sigma_e^{y_1} & 0 \\ 0 & \sigma_e^{y_0} \end{array}\right]\left[\begin{array}{c} \tilde{e}_t^{y_1} \\ \tilde{e}_t^{y_0} \end{array}\right]. \tag{2.11}$$

Therefore, I allow the output expectation shocks to be dynamically correlated. This assumption allows them to depend on each other, but it preserves the interpretation of each as an identifiable structural shock, as the variance-covariance matrix is still assumed to be diagonal. This structure, however, is not crucial, as one could, instead, assume contemporaneous correlation in the innovations and then impose an identification condition, for example by assuming recursiveness, to compute the impulse responses and the variance decomposition.

Expectations shocks are, therefore, identified as the exogenous component of expectations that is not related to current fundamentals and not accounted for by the learning model. Expectation shocks are orthogonal to current fundamentals and to the near-rational expectations about future fundamentals formed from the learning model. Data on expectations are exploited to provide information on the best-fitting learning process over the sample and to disentangle the part of expectations that is due to an endogenous response to the state of the economy and the exogenous expectation shock.

The intuition regarding the expectations formation works as follows. Agents usually form expectations in a near-rational way, by using past values of economic variables and their most recent beliefs about the structure of the economy to forecast future macroeconomic variables. But agents may deviate from these near-rational forecasts: they can be either more optimistic – by believing that future output will be higher than predicted by their learning model (or inflation lower) – or more pessimistic. One of the main goals of the paper will be to evaluate the role and empirical importance of these estimated exogenous waves of optimism and pessimism.

Economic agents base their optimizing decisions on $t-1$ information. They observe the values of endogenous variables up to $t-1$ and they update their beliefs through (2.5) and (2.6) running regressions of the endogenous variables in $t-1$ on a vector of intercepts and on the variables in $t-2$; they can then form expectations about variables in t and $t+1$.

The model abstracts from any issue related to the heterogeneity of forecasts. The expectations that are relevant in the model are averages across forecasters (the degree of heterogeneity does not enter the model).[5] In the same way, while it is likely that agents with various degrees of optimism and pessimism coexist and interact in every period, the expectation shock that enters the model is intended as an indication of the aggregate mood of the market.

2.2. **Structural and Expectations Shocks.** The economy is therefore subject to a combination of structural and expectational shocks. The structural shocks are related to aggregate demand (the natural rate disturbance r_t^n), supply (the cost-push shock u_t), and policy (the monetary policy shock ε_t). The expectation shocks, instead, are $e_t^{y_0}$, $e_t^{y_1}$, e_t^{π}, and e_t^{i}, and affect the formation of expectations about future real activity, future inflation, and future policy changes. The shocks are assumed to be contemporaneously uncorrelated, but the expectations shocks about output at different horizons $e_t^{y_0}$ and $e_t^{y_1}$ are allowed to be dynamically correlated.[6]

3. Near-Rational Expectations Econometrics: Estimation Approach

3.1. **Realized and Expectations Data.** I exploit available data on expectations, along with realized data on macroeconomic variables, to estimate the structural parameters of the model, to infer the economic agents' learning process over the sample, and to identify the expectations shocks. The expectations data are derived from the Survey of Professional Forecasters (SPF) when possible. I use the mean across forecasters regarding the one-period-ahead expected Nominal GDP (acronym NGDP) and the one-period-ahead expected Price level (acronym PGDP).[7] Expectations about real

[5] An exploration of the role of heterogeneous forecasts on the business cycle is provided by Branch and McGough (2009).

[6] The assumption of zero correlation is standard in the literature estimating DSGE models (an exception if Curdia and Reis, 2009). I have found that the assumption of zero correlation between the remaining expectational shocks and between the expectational shocks and the structural shocks is substantially satisfied looking at ex-post results. In the robustness section, however, I will show that the results are robust to allowing a non-zero correlation between u_t and r_t^n, for example.

[7] One potential problem with this approach is that by using the mean across forecasters, we may confound actual variation in expectations over time with a "composition effect", which is, instead, due to the changing composition

GDP are constructed by dividing the expected Nominal GDP data by the expected GDP Price Deflator from the survey. Expected inflation is calculated as the log of the expected two-quarter-ahead GDP Price Deflator minus the log of the expected one-quarter-ahead GDP Price Deflator (both expectations are formed in t with an information set that includes values of the variables up to $t-1$).

Households' optimality conditions also require them to form expectations about one-period-ahead nominal interest rates. Such expectations are available from the SPF, but only starting from 1981:III. In the baseline estimation in the paper, however, I choose to exploit the longest possible sample and, hence, I derive expectations about future interest rates using the expectations theory of the term structure. This implies the relation $i_t^{6M} = \left(\frac{i_t^{3M}+\widehat{E}_t i_{t+1}^{3M}}{2}\right) + \bar{\varsigma}$, which states that the six-month yield is equal to the average between the current three-month yield and the expected three-month yield three months from now, except for constant term premium $\bar{\varsigma}$, and which can be solved for $\widehat{E}_t i_{t+1}^{3M}$ at each t. Data on the three-month Treasury bill rate are used for i_{t-1}^{3M} and on the six-month Treasury bill rate for i_{t-1}^{6M}. The estimation will also be repeated using the expected interest rate from the SPF and the shorter post-1981 sample as a robustness check. The correlation between forecasts derived from the expectations theory and forecasts from the SPF, in the period in which they are both available, is equal to 0.978.

To better explain the observed expectations and to more accurately identify the economic agents' learning process, it is desirable to exploit knowledge about their real-time information set. Such information is fortunately available from the SPF, since, in each quarter t, when forecasters receive the survey, they are asked about their perceptions about the values of the variables in $t-1$. When forming their expectations about variables in $t+1$, therefore, they use their best estimates for the variables in $t-1$, which are also available from the SPF. Moreover, one week before the survey is mailed to the forecasters, the BLS releases data about the values of the variables in $t-1$. Almost all forecasters in the survey simply report the BLS release as their perception of $t-1$ values.[8] Forecasters observe the values of the variables in $t-1$ when communicating forecasts for variables dated t and $t+1$. The forecasters' $t-1$ information set is available to the econometrician and, therefore, will be exploited in the estimation.

Real GDP is constructed using the real-time Nominal GDP series divided by the real time GDP Implicit Price Deflator, using the data from the SPF regarding the $t-1$ information set available to agents, i.e. using the BLS real-time data release about GDP. Inflation is constructed using real-time data on the price deflator (acronym PQvQd), obtained from the 'Real Time Data Set

of the set of individual forecasters (e.g., Mansky, 2010). At the aggregate macroeconomic level, however, this effect may be expected to be small.

[8] *Survey of Professional Forecasters, Documentation, February 2010, update.*

for Macroeconomists', made available by Federal Reserve Bank of Philadelphia and described in Croushore and Stark (2001). As short-term nominal interest rate, I use the three-month Treasury bill (I choose the three-month Treasury bill, as forecasts data from the SPF will also be available for this variable).[9]

The realized and expectations series regarding inflation, the growth rate of output, and interest rates, are shown in Figure 1. As known, forecasts about inflation underestimate actual inflation for a large part of the 1970s, particularly in correspondence of its first peak, while they often overestimate inflation in the second part of the sample. Expectations about inflation and output growth are substantially smoother than the actual series.

3.2. **State-Space System.** The model, summarized by equations (2.1)-(2.3), and with expectations formed as in (2.7)-(2.8), can be written in state-space form as

$$\xi_t = A_t + F_t \xi_{t-1} + G w_{t-1} \tag{3.1}$$

$$\Upsilon_t = H \xi_t + \Delta_0 + \Delta_1 t \tag{3.2}$$

where the state vector ξ_t includes the endogenous variables, expectations, as well as structural and expectational disturbances: $\xi_t = \left[y_t, \pi_t, i_t, r_t^n, u_t, \widehat{E}_{t-1} y_{t+1}, \widehat{E}_{t-1} \pi_{t+1}, ..., e_t^{y_1}, e_t^{y_0}, e_t^{\pi}, e_t^{i} \right]'$. Equation (3.1) represents the transition equation for the states, while equation (3.2) is the measurement equation, which relates the available observable variables to the state vector. The matrix H_t is a matrix of zeros and ones, which simply selects the observables Υ_t from the state vector. The small-scale state space for the New Keynesian model, therefore, sizeably expands due to the use of expectations and the inclusion of expectational shocks. The model is estimated to match the following observable variables: output, inflation, short-term interest rate, output forecasts (one-period-ahead), output forecasts (two-period-ahead), inflation forecasts (two-period-ahead), interest rate forecasts (one-period-ahead).

The baseline estimation assumes a linear trend for output. The estimation is, however, repeated under a variety of trend and potential output specifications, which are discussed later in the paper (e.g., using the potential GDP measure from the CBO, or the theoretically-consistent output gap). The choice of using an output measure based on a linear trend as the benchmark case aims to capture the real-time forecasting process of actual economic agents over the whole sample: a deterministic trend is likely to be a better approximation of the detrending procedures that forecasters had in mind for a large portion of the sample than the theoretical New Keynesian definition of the output gap. Moreover, a deterministic time trend makes it possible to consider learning about trend by economic agents in a simple and transparent way. I assume, in fact, for now, that economic agents

[9] In principle, one could use market expectations extracted from the Federal Funds rate future contract as an observable, instead, but data are available only from 1988.

know the parameters in the trend equation, i.e. Δ_0 and Δ_1 in (3.2) (this is still in the spirit of considering a minimal deviation from the rational expectations case, under which they would perfectly observe the trend). But I will later relax this assumption and allow agents to learn about the coefficients in the trend equation as well.

3.3. **Priors.** Table 1 reports information about the prior distributions. There is large uncertainty on the value of the intertemporal elasticity of substitution coefficient σ. Values used in macro studies, as well as estimates from micro data, range from values very close to 0 to values substantially above 1 (e.g., Hall, 1988, Gruber, 2006, Woodford, 2003). I choose a Gamma prior with mean 1 and a rather large standard deviation equal to 0.75. For the slope of the Phillips curve coefficient κ, I assume a Gamma prior distribution with mean 0.25 and standard deviation equal to 0.177.

Regarding the monetary policy rule, the feedback coefficients χ_π and χ_y follow Normal prior distributions with mean 1.5 and standard deviation 0.25 and mean 0.25 and standard deviation 0.125, while the interest-rate smoothing coefficient ρ is assumed to follow a Beta distribution with mean 0.8.

The autoregressive coefficients in the AR processes for structural and expectational disturbances are assumed to follow Beta prior distributions with mean 0.5 and standard deviation 0.26. The chosen distributions are almost non-informative: they assign relatively uniform probabilities to all values of the autoregressive coefficients, except for extreme values close to zero or one, which are downplayed. Inverse Gamma distributions are chosen for the standard deviations of the shocks.

An important parameter in the estimation is the constant gain coefficient. To minimize the influence of prior information and of assumptions about the learning process, I assume a non-informative Uniform prior distribution for the gain over the [0,1] interval.

3.4. **Bayesian Estimation.** The model is estimated using Bayesian methods over the 1968:IV-2009:I sample. The starting date coincides with the first quarter of availability of the survey forecasts. The likelihood of the state-space system (3.1)-(3.2) is derived using the Kalman filter.

The Metropolis-Hastings algorithm is used to generate draws to approximate the posterior distribution. I run 500,000 draws, discarding an initial burn-in given by the first 25% draws. Convergence is evaluated by looking at trace plots, CUSUM plots, and performing the tests proposed by Geweke (1992) and Raftery and Lewis (1995); I use bivariate scatter plots to assess the mixing of the chain and to check whether strong dependence exists among some parameters.

Rather than imposing a learning process a priori and obtaining results that are conditional on a given learning process, I also estimate the learning parameters jointly along with the other structural parameters of the economy. In particular, both the constant gain coefficient and the uncertainty that characterizes agents' initial beliefs, i.e. the variance-covariance matrix $R_{t=0}^{-1}$, are inferred from the estimation. The initial precision matrix $R_{t=0}$ is given by $R_{t=0} = \left[\overline{\mathbf{g}} \sum_{i=1}^{\tau} (1-\overline{\mathbf{g}})^{(i-1)} X_{\tau-i} X'_{\tau-i}\right]$, where τ indexes the pre-sample observations; therefore, by estimating a single coefficient, the constant gain $\overline{\mathbf{g}}$, the estimation provides evidence both on the learning speed and on the uncertainty surrounding initial beliefs. In this way, the best-fitting learning process can be extrapolated from time series data. This paper improves over previous work on the estimation of general equilibrium models with learning (e.g., Milani, 2007a,b, 2008) by exploiting actual data on expectations to best infer the evolution of the learning process over the sample.

Pre-sample data for the 1947:I-1968:IV period are used to inform the choice of initial beliefs in the learning algorithm. The initial values related to the inflation law of motion are characterized by a moderate perceived persistence in inflation ($b_{11} = 0.7$), by a negative intercept in the inflation equation ($b_{10} = -0.05$), and by a perceived sensitivity of inflation to output equal to 0.001. In the output equation, the initial values point to a larger degree of output persistence ($b_{22} = 0.8$), and to a sensitivity of output to changes in the real interest rate equal to $b_{24} = -0.1$. The initial beliefs about the perceived monetary policy rule coefficients indicate a modest reaction to inflation and output, and a degree of interest rate inertia equal to 0.75. In the robustness section, I will discuss how the empirical results do not depend on a particular choice of initial beliefs.

4. Near-Rational Expectations Econometrics: Empirical Results

4.1. **Posterior Estimates.** The posterior estimates for the baseline model are presented in Table 2. The table shows the posterior mean and 95% credible intervals for each estimated parameter. The estimates indicate a posterior mean for the sensitivity of inflation to output κ equal to 0.035 and for the elasticity of intertemporal substitution σ equal to 0.236. The estimates of the monetary policy rule coefficients are consistent with the vast majority of previous studies: they indicate a large degree of policy inertia ($\rho = 0.95$) and reaction coefficients to inflation and output equal to 1.417 and 0.221.

The data are informative about the best-fitting learning process in the sample. The posterior estimate for the constant gain parameter is equal to 0.0196, with a 95% credible interval between 0.015 and 0.025 (obtained under a non-informative Uniform prior between 0 and 1). This estimate, which is obtained by fitting the learning process to expectations data, is remarkably similar to the estimate found in Milani (2007a) in an estimation that used information on realized variables only (that paper found $\overline{\mathbf{g}} = 0.0187$). The evidence, therefore, points to values of the gain close to 0.02, which have often been used as benchmark in simulated learning models, as the most empirically realistic.

The main interest of the paper lies in identifying the effects of structural and expectational shocks. Regarding the structural shocks, the posterior estimates suggest a relatively low persistence of these disturbances. The autoregressive coefficients have mean equal to 0.351 for the demand shock r_t^n and to 0.171 for the supply shock u_t. Since the model lacks "mechanical" sources of persistence as habit formation and inflation indexation and, yet, the structural shocks are characterized by a substantially lower persistence than usually obtained in these models, the estimation suggests that the inclusion of observed expectations and learning can successfully induce realistic levels of persistence in the system. The main novelty in the estimation lies in identifying the expectational shocks. These are found to be generally quite persistent. The autoregressive coefficients have posterior means equal to 0.854 for $e_t^{y_1}$, 0.422 for e_t^{π}, and 0.627 for e_t^{i}, while the coefficient is smaller for $e_t^{y_0}$ ($\rho_e^{y_0} = 0.231$). I have allowed the expectational disturbances related to output expectations to be dynamically correlated: I find that $e_t^{y_0}$ is strongly connected to the previous period $e_t^{y_1}$ ($\rho_{y_0,y_1} = 0.722$, while ρ_{y_1,y_0} is basically zero). This means that the shock related to expectations formed in $t-1$ about output in t is closely connected to the previous shock related to the longer-horizon expectations formed in $t-2$ about output in t.

The evolution of the estimated beliefs over the sample is shown in Figure 2. Economic agents revise their beliefs about the behavior of inflation over time: they increase their estimate of inflation persistence from below 0.65 around 1970 to peaks above 0.8 and then around 0.75 later in the 1970s. Their perceived level of steady-state inflation also increases in the 1970s before declining in the second part of the sample. In the output gap equation, the perceived persistence of the output gap increases over the sample, while the perceived sensitivity of the gap to interest rates becomes larger in the 1970s, but substantially smaller in the 1980s and 1990s. The estimation also provides evidence on the market beliefs about the monetary policy rule coefficients over the sample. These beliefs reveal that the market began to expect higher average rates since the early 1980s, as evidenced by their intercept estimate, that the Fed's reaction coefficient toward inflation has been perceived to jump around 1979, while the reaction coefficient toward output has been revised in the opposite direction in the 1980s. The private agents also recognize a shift in the degree of interest rate inertia that also takes place around 1980.

4.2. **Expectation Shocks as Drivers of Economic Fluctuations.** I derive impulse response functions for the macroeconomic variables in the model to both structural and expectation shocks. The impulse responses are derived using the last 10,000 draws from the MCMC. The impulse responses in the figures denote averages over the sample and across draws and are shown along with their respective 2.5 and 97.5% percentiles.

Figure 3 overlaps the impulse responses of detrended output to the aggregate demand natural rate shock r_t^n and to the output expectation shock $e_t^{y_1}$, which can be interpreted as an "optimism" shock.

The effect of the structural demand shock is rather large on impact, but its transmission is relatively quick: the peak of the output response already occurs in the second quarter after the shock. The expectation shock, instead, leads to a much more persistent output response. The peak occurs after a year and a half and the effect is larger and more long-lived than the effect of the structural shock.

Figure 4 shows the response of inflation to cost-push and natural rate shocks, along with the response to expectational shocks regarding future inflationary pressures and real activity. The supply shock and the expectational shock regarding future inflation die off rather quickly, in slightly more than a year. Demand shocks induce a more inertial adjustment. In particular, the response of inflation to the output expectation shock is more pronounced and sluggish. The expectational shock $e_t^{y_1}$ (and $e_t^{y_0}$ as well), therefore, resembles a demand shock, as an increase in optimism about the future state of the economy moves output and inflation in the same direction.

The model specification also permits to study the determinants of observed private agents' expectations. Figure 5 displays the impulse responses of the observed expectation variables to the corresponding structural demand and expectational shock about future output ($e_t^{y_1}$), for example. Expectational shocks induce a larger and more and persistent response in expectations than fundamental shocks to the natural rate (this is true in the case for belief shocks about future output, shown in the figure, but expectations about future interest rates, for example, respond more to unexpected shocks to the Taylor rule than to expectational shocks about future policy rates).

Table 3 reports the outcome of the forecast error variance decomposition: the table shows the mean shares across the last 10,000 MCMC draws, along with the 2.5 and 97.5 percentiles, for different horizons (equal to 4, 12, and 20 quarters) that should allow to capture the importance of shocks at business cycle frequencies.

Expectation shocks regarding future output are the main source of economic fluctuations. These expectational shocks can account, in fact, for 53-54% of fluctuations (at horizons equal to 12 and 20 quarters). Natural rate shocks explain around 20%, monetary policy shocks 22%, and cost-push shocks 3% of fluctuations. Structural shocks are, however, more important in the very short run: natural rate shocks account for 41% of fluctuations at the one year horizon and for a larger share at even shorter horizons.

Expectational shocks are also the main contributor to the variability of inflation. Cost-push shocks explain 27% of its variance (at the 20 quarters horizon), while expectation shocks regarding future inflation explain 17% and expectation shocks regarding future real activity explain 33%. The role of structural cost-push shocks is again larger for fluctuations within the one-year horizon (for which, they arrive at a share of 50% or more).

The posterior densities of the total shares of the output gap, inflation, and interest rates variances that are explained by the structural demand shock r_t^n and by the expectation shocks $e_t^{y_1}, e_t^{y_0}$ are shown in Figure 6 (for a longer horizon of 40 quarters). As apparent from the figure, the estimation assigns very large probability to the conclusion that expectation shocks are more important than the conventional structural shocks that affect aggregate demand in the New Keynesian model in driving business cycle fluctuations. Expectational shocks about future real activity are also more important than unexpected demand shocks in explaining the dynamics of inflation and nominal interest rates, although in the latter case there is a large uncertainty on the extent of their contribution.

The expectation shock regarding future output is shown in Figure 7, along with vertical bands denoting NBER recession dates. The figure shows that expectation shocks quickly fall during recessions and become negative, indicating an increasing aggregate pessimism (that is, economic agents form expectations about future economic conditions that fall below what their near-rational model and their updated beliefs would suggest). The expectation shock begins to fall right before the economy enters a recession and increases before the recession ends. The degree of optimism is usually at the highest in the middle of an expansion.[10]

Expectations are affected by developments in the economy. Fundamental shocks can explain roughly one third of expectation data regarding output, inflation, and interest rates. But expectations are mostly driven by expectational innovations. Purely expectational shocks, in fact, account for roughly 60% of the variance in output, inflation, and interest rate expectations.

It is important to point out that the results on the importance of expectation shocks do not arise from a serious misspecification of the learning model or from its failure to match expectations data. Figure 8 plots the agents' near-rational expectations derived from the learning model's PLM versus the observed survey forecasts. The series track each other remarkably well. The worst fit is observed in the case of inflation, but it is still very satisfactory over the sample, with the possible exception of the 1976-1977 observations in which the learning model would imply a downward revision in inflation expectations, which, instead, does not materialize in actual expectations data (it is worth noting that the learning model would be correct, as realized inflation was actually lower than expected, as can be seen from Figure 1). The importance of expectation shocks stems in part from the result that the exogenous waves of optimism and pessimism appear quite persistent over

[10]From the figure it seems that agents are on average more optimistic in the first part of the sample than in the second. This feature, however, is not robust to the use of the alternative output gap measures that will be discussed in section 5. The dynamics of the expectation shock over recessions and expansions is, instead, consistent across different specifications.

time. While structural shocks may have large effects on the economy in the short run, they are here not strongly serially correlated and their transmission has been shown to be rapid. Shifts in market sentiments, instead, take a long time to reverse direction.

The empirical results may be taken to suggest that macroeconomic model building should be revisited. A substantial modeling effort is directed toward incorporating frictions in current DSGE models to match the sluggish response of macro variables to structural shocks. But this paper suggests that the response to aggregate demand and supply shocks may be faster than commonly thought. Under the paper's framework, sluggishness in the economy is, instead, induced by learning and by slow-moving expectational shifts.

4.3. **The Role of Learning.** To quantify the role of endogenous changes in expectations through learning, rather than exogenous changes, in contributing to economic fluctuations, let's now assume that the PLM used by economic agents corresponds exactly the same as it would be under rational expectations (this would be the case if the learning process would have already converged to the rational expectations equilibrium):

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \overline{a} + \overline{b} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + \overline{c} r^n_{t-1} + \overline{d} u_{t-1} + \epsilon_t. \tag{4.1}$$

Economic agents, therefore, are now assumed to have knowledge not only about the structural form of the minimum state variable solution, but also about all the coefficients in the model and about the values of the structural disturbances (maintaining the $t-1$ information set). The coefficients are now constant over the sample (agents have already learned the truth) and the intercept vector is recognized to contain only zeros.

I re-estimate the model, now endowing private agents with the correct model of the economy, i.e. equation (4.1), and the correct coefficient values $\overline{a}$, $\overline{b}$, $\overline{c}$, and $\overline{d}$. This case mimics the situation of rational expectations. The expectation shocks are identified as the part of the observed survey and market expectations that deviates from the forecasts implied by the rational expectations PLM. I compute the variance decomposition under this new scenario.

Under the rational expectations PLM, the expectational shocks now account for a larger share of fluctuations: the expectational shock about output accounts for 89% of output fluctuations, while the expectational shock about future inflation accounts for 62% of inflation fluctuations. The results may be taken to indicate that out of 89% of output gap fluctuations that may be attributed to expectational shocks if the economy had already converged to the rational expectations equilibrium and if agents used the corresponding RE-PLM, 40% may be rationalized as the endogenous response due to learning in relation to an evolving state of the economy, while the remaining 50%, as found before, is due to exogenous expectation shocks, which are unrelated to fundamentals.[11] In the case of inflation, out of 62% of expectation shocks regarding future inflationary pressures, the most part ($\approx 45\%$) is possibly due to a near-rational response to changes in the economy and to learning.

5. Robustness

5.1. **Assumptions about Trend and Potential Output.** The results have been so far presented under the baseline case, which assumes that the output gap is approximated empirically by a linearly detrended measure of output. This measure, however, doesn't correspond to the theoretical definition of the output gap in the model, which would be given by the deviation of output from its corresponding level in the same economy, but under flexible prices. Therefore, it is necessary to evaluate the robustness of results to different assumptions about the output gap, including using the theoretical definition.

The results do not hinge on the choice of a peculiar output gap measure: all conclusions are robust to a wide variety of detrending options and to different ways to characterize potential output.

I do not discuss here the results under other deterministic time trends (e.g., quadratic, segmented), as they are absolutely similar to the baseline case. It is, instead, worth relaxing the assumption that agents know the coefficients in the trend equation (i.e., Δ_0 and Δ_1), by allowing them to learn about the trend as well over time (I assume that they learn about the trend using a decreasing gain t^{-1}). So far, their misperceptions about the trend were captured only by the intercept term in the learning rule. From the estimation, economic agents appear the overestimate the growth rate of output in the 1970s, and underestimate it later on, before converging to the correct Δ_1 at the end of the sample.

As an alternative, the output gap can be constructed as the log deviation of real GDP from the CBO's estimate of potential GDP. Therefore, I repeat the analysis using the growth rate of real GDP, expectations about the growth rate of real GDP between t and $t+1$ and about the growth rate of real GDP between $t-1$ and $t+1$ as the observable variables (besides inflation, expected inflation, interest rates, and expected interest rates) that should be matched in the estimation, and assuming that agents know the growth rate of potential GDP, which is taken from the CBO series (I have also checked the case in which they learn about the growth rate using an AR(1) learning rule with similar results). Real-time data on potential GDP could also be used in the estimation, but unfortunately they exist only starting from 1991.

[11]There is an important caveat here. The results are obtained assuming that the economy is characterized by the same model that was used under learning. In this scenario, expectation shocks become even more important as expectations derived from the new PLM fall very far from the observed expectations. But if the model was re-estimated under rational expectations (shutting down the expectation shocks), the structural coefficients would likely change in the effort of matching the endogenous variables in the new framework and the results might significantly differ.

Finally, the model is re-estimated using the theoretical definition of the output gap, i.e. the deviation of output from its flexible-price level. The model is estimated again using growth rates of real GDP, and expectations about the growth rates from the SPF as observables, and the implied output gap in the model is obtained from the filtering procedure.

Table 4 shows the results from the variance decomposition obtained in each of the previous cases. Expectational shocks referring to future output gaps are always the dominant source of fluctuations: their share goes from 48% to almost 70%. The share of the natural rate shock is smaller, with posterior means below 20%.

5.2. **Correlated Shocks.** Since any deviation of the output gap from its theoretical definition will materialize in the shocks, u_t and r_t^n may be correlated. The estimation is repeated allowing them to be correlated: the conclusions from the variance decomposition, as shown in Table 4, are unchanged.

5.3. **Different Learning Speeds and Alternative Initial Beliefs.** It is likely that economic agents learn about the dynamics of different macroeconomic variables at various speeds. This section allows the gain coefficients to differ across variables. The posterior means for the constant gain related to inflation and output are equal to 0.0179 and 0.0296. Learning about the Federal Reserve's policy rule has been slower over the sample: the posterior mean for $\bar{\mathbf{g}}_i$ is 0.005. The implied impulse responses and variance decomposition, however, remain similar to those in the baseline case.

The estimation has been repeated under a variety of combinations for the initial values of agents' beliefs, by assuming, for example, different sensitivities of output to interest rates (with a range of b_{yi} from 0 to -1), of inflation to the output gap and interest rates ($b_{\pi y}$ from a minimum of 0 to a maximum of 0.2 and $b_{\pi i}$ from a maximum of 0.2 to a minimum of -1, respectively), of interest rates to the output gap and inflation ($b_{iy}, b_{i\pi}$ from 0 to 0.2), different intercepts (from -0.5 to 0.1 for inflation, from 0.2 to -0.2 for output, and from -0.2 to 0.2 for interest rates), and different autoregressive coefficients (from 0.4 to 0.9 in the inflation equation, from 0.8 to 0.95 in the output gap equation, and from 0.65 to 0.95 in the interest rate equation). While different initial beliefs obviously imply a somewhat different evolution of beliefs in the first part of the sample and, in some cases, affect the estimate of the constant gain coefficient, the outcomes regarding the share of fluctuations due to expectational shocks (and about the expectational shock that refers to future output) in the variance decomposition are always robust. Table 4 reports the outcome under one particular case.

5.4. **Observed Structural Disturbances in PLM.** In the baseline estimation, I have assumed that economic agents learn using a PLM that corresponds to a VAR(1) in the endogenous variables in the model. This is arguably the most empirically realistic case, since the unobserved structural disturbances are assumed to be outside the agents' information set. The theoretical literature on learning, however, typically assumes that agents can observe the structural shocks as well. I verify here the robustness of the results to assuming that agents use a PLM that has exactly the same structural form of the model solution under rational expectations: the disturbances are now observed by the agents (given the $t-1$ information set, I assume that in period t they observe shocks in $t-1$, but not those in t, although this is unimportant), while the reduced-form coefficients are unknown and need to be inferred from historical time series data. This case corresponds to an even more minimal deviation from rational expectations.

The posterior estimates remain similar: the gain coefficient, for example, has a mean equal to 0.0181, only slightly lower than before. Expectation shocks regarding future real activity are confirmed to be the main driver of economic fluctuations: they explain 57% of the output gap variability and compete with cost-push shocks as the main determinant of inflation movements.

5.5. **SPF Interest-Rate Expectations and Post-1981 Sample.** While data on expectations about output and inflation were obtained from the SPF, in the baseline estimation I have extracted interest rate forecasts from the term structure of interest rates, by assuming that the expectation theory of the term structure holds. This choice was motivated by the intention of using the longest possible sample in the benchmark estimation. Data on expectations about future interest rates (corresponding to $\widehat{E}_{t-1}i_t$ in the model), however, are also available from the SPF, but starting from 1981:III. I can now repeat the estimation for the post-1981 sample and using SPF forecasts for all series.

The posterior estimates indicate that the structural disturbances are even less persistent in this sub-sample ($\rho_u = 0.081$, $\rho_r = 0.147$); the same is true, although to a minor extent, for expectational shocks (e.g., $\rho_e^{y_1} = 0.714$, $\rho_e^{y_0} = 0.288$). The standard deviations for the structural and expectational innovations are lower than their full-sample counterparts. The constant gain is also lower after 1981 ($\mathbf{\bar{g}} = 0.009$).

The main conclusions are unchanged. The role of expectation shocks regarding future monetary policy choices on the business cycle is confirmed to be small: this outcome is robust to the use of implied interest rate forecasts from the term structure or survey forecasts.

Moreover, the results indicate that expectation shocks were not only important in the 1970s, but they also represent the main source of output fluctuations in the more stable post-1981 period. Expectational shocks about future output explain roughly 60% of fluctuations, while natural rate and monetary policy shocks add to explain around 30%. The main difference between the pre- and post-1981 periods seems that unexpected monetary policy shocks were considerably more important in the first subperiod.

5.6. **TV Monetary Policy Coefficients.** The baseline model assumed constant gain learning, but it didn't incorporate any actual source of variation in the model coefficients. This has been assumed merely for simplicity. This section evaluates the robustness of the results to this assumption. The model is re-estimated under the assumption that there is a structural break in the monetary policy rule coefficients in correspondence of the start of Volcker's tenure as Chairman of the Federal Reserve, which is not known by private agents. The estimates indicate that the reaction coefficient to inflation increases from around 1 to 1.93 and the reaction coefficient to the output gap declines from 0.33 to close to 0. As shown in Table 4, the results of the paper are, however, robust to the assumption of a time-varying monetary policy rule, as expectation shocks still explain roughly half of output fluctuations.

6. Conclusions

While economists have recognized for a long time that psychological forces, changes in market sentiments, shifts in confidence, and so forth, may exert a large influence on economic fluctuations, the current generation of macroeconomic models typically excludes them from the analysis.

This paper argues that these forces, in the form of exogenous expectational shifts, such as waves of optimism and pessimism, should be brought back to the center of macroeconomics.

The paper has estimated a baseline New Keynesian model and exploited observed survey data on expectations or expectations extrapolated from the market. In this way, the paper has allowed a departure from the conventional rational expectations hypothesis, which is widespread in macroeconomics. The observed expectations were assumed to be formed, instead, from a near-rational mechanism. Economic agents, however, were allowed to deviate each period from the forecasts that were implied by their learning model and that were hence justified as an endogenous response to the state of the economy. The deviations are captured by expectation shocks in the model.

The empirical evidence has shown that expectational shocks, particularly those related to future real activity, may play a large role in driving the business cycle. These shocks can explain half of economic fluctuations over the sample.

There are some limitations that should be dealt with in future research. In particular, it is recognized that a more definitive answer on the importance of expectation shocks versus alternatives as technology and demand shocks would require moving to a larger-scale model. This paper, however, aims to provide some initial empirical evidence in support of the potential importance of expectational factors, which are often disregarded in empirical analyses, as an important autonomous driver of the business cycle. The results indicate that further research to evaluate their importance is necessary.

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Prior Distributions				
Descr.	Param.	Distr.	Mean	95% Prior Int.
Slope PC	κ	Γ	0.25	[0.03-0.7]
IES	σ	Γ	1	[0.1-2.92]
IRS	ρ	B	0.8	[0.46-0.98]
Feedback Infl.	χ_π	N	1.5	[1.01-1.99]
Feedback Gap	χ_y	N	0.25	[0.01-0.49]
Autoregr. Demand shock	ρ_r	B	0.5	[0.05-0.95]
Autoregr. Cost-push shock	ρ_u	B	0.5	[0.05-0.95]
Std. Demand shock	σ_r	Γ^{-1}	0.5	[0.1-1.92]
Std. Cost-push shock	σ_u	Γ^{-1}	0.5	[0.1-1.92]
Std. MP shock	σ_ε	Γ^{-1}	0.5	[0.1-1.92]
Autoregr. Exp. shock y_{t+1}	$\rho_{e^{y_1}}$	B	0.5	[0.05-0.95]
Autoregr. Exp. shock y_t	$\rho_{e^{y_0}}$	B	0.5	[0.05-0.95]
Autoregr. Exp. shock π_{t+1}	ρ_{e^π}	B	0.5	[0.05-0.95]
Autoregr. Exp. shock i_t	ρ_{e^i}	B	0.5	[0.05-0.95]
Depend. $e_t^{y_1}$ on $e_{t-1}^{y_0}$	ρ_{y_1,y_0}	N	0	[-0.98-0.98]
Depend. $e_t^{y_0}$ on $e_{t-1}^{y_1}$	ρ_{y_0,y_1}	N	0	[-0.98-0.98]
Std. Exp. shock y_{t+1}	$\sigma_{e^{y_1}}$	Γ^{-1}	0.5	[0.1-1.92]
Std. Exp. shock y_t	$\sigma_{e^{y_0}}$	Γ^{-1}	0.5	[0.1-1.92]
Std. Exp. shock π_{t+1}	σ_{e^π}	Γ^{-1}	0.5	[0.1-1.92]
Std. Exp. shock i_t	σ_{e^i}	Γ^{-1}	0.5	[0.1-1.92]
Constant gain	$\overline{\mathbf{g}}$	U	0.5	[0.025-0.975]

Table 1 - Prior distributions.

Note: Γ denotes Gamma distribution, B denotes Beta distribution, N denotes Normal distribution, Γ^{-1} denotes Inverse Gamma distribution, and U denotes Uniform distribution.

		Posterior Distribution	
Descr.	Param.	Mean	95% Credible Interval
Slope PC	κ	0.035	[0.019-0.053]
IES	σ	0.236	[0.03-0.55]
IRS	ρ	0.95	[0.91-0.98]
Feedback Infl.	χ_π	1.417	[0.97-1.86]
Feedback Gap	χ_y	0.221	[0.06-0.43]
Autoregr. Demand shock	ρ_r	0.351	[0.19-0.50]
Autoregr. Cost-push shock	ρ_u	0.171	[0.04-0.31]
Std. Demand shock	σ_r	0.77	[0.69-0.86]
Std. Cost-push shock	σ_u	0.297	[0.27-0.33]
Std. MP shock	σ_ε	0.207	[0.19-0.23]
Autoregr. Exp. shock y_{t+1}	$\rho_e^{y_1}$	0.854	[0.68-0.98]
Autoregr. Exp. shock y_t	$\rho_e^{y_0}$	0.231	[0.08-0.41]
Autoregr. Exp. shock π_{t+1}	ρ_e^{π}	0.422	[0.28-0.56]
Autoregr. Exp. shock i_t	ρ_e^{i}	0.627	[0.51-0.74]
Depend. $e_t^{y_1}$ on $e_{t-1}^{y_0}$	ρ_{y_1,y_0}	-0.009	[-0.13-0.15]
Depend. $e_t^{y_0}$ on $e_{t-1}^{y_1}$	ρ_{y_0,y_1}	0.722	[0.50-0.91]
Std. Exp. shock y_{t+1}	$\sigma_e^{y_1}$	0.286	[0.26-0.32]
Std. Exp. shock y_t	$\sigma_e^{y_0}$	0.342	[0.30-0.38]
Std. Exp. shock π_{t+1}	σ_e^{π}	0.203	[0.18-0.23]
Std. Exp. shock i_t	σ_e^{i}	0.087	[0.08-0.10]
Constant gain	$\overline{\mathbf{g}}$	0.0196	[0.015-0.025]

Table 2 - Posterior Estimates, baseline model.

Note: The table shows posterior means and 95% credible intervals calculated over 500,000 Metropolis-Hastings draws, discarding an initial burn-in of 25% draws. The sample is 1968:III-2009:I.

	π_t	y_t	i_t	$\widehat{E}_{t-1}\pi_{t+1}$	$\widehat{E}_{t-1}y_{t+1}$	$\widehat{E}_{t-1}y_t$	$\widehat{E}_{t-1}i_t$
$h = 4$							
Cost-Push Shock u_t	0.507 [0.42,0.61]	0.022 [0.01,0.03]	0.02 [0,0.06]	0.233 [0.18,0.32]	0.031 [0.02,0.05]	0.025 [0.01,0.04]	0.019 [0.01,0.05]
Natural Rate Shock r_t^n	0.05 [0.03,0.09]	0.413 [0.32,0.51]	0.011 [0,0.03]	0.044 [0.03,0.07]	0.262 [0.20,0.34]	0.366 [0.28,0.45]	0.011 [0,0.02]
MP Shock ε_t	0.05 [0.04,0.07]	0.083 [0.06,0.12]	0.943 [0.87,0.99]	0.08 [0.06,0.10]	0.092 [0.07,0.12]	0.077 [0.06,0.11]	0.824 [0.76,0.88]
Expect. Shock e_t^π	0.317 [0.23,0.39]	0.01 [0,0.02]	0.013 [0,0.04]	0.588 [0.50,0.66]	0.014 [0.01,0.03]	0.01 [0,0.02]	0.012 [0.01,0.03]
Expect. Shock e_t^{y1}	0.05 [0.02,0.09]	0.410 [0.31,0.50]	0.01 [0,0.013]	0.033 [0.02,0.05]	0.526 [0.43,0.61]	0.420 [0.33,0.50]	< 0.01 [0,0.009]
Expect. Shock e_t^{y0}	0.013 [0.01,0.02]	0.050 [0.03,0.08]	< 0.01 [0,0.01]	0.011 [0.01,0.02]	0.063 [0.03,0.10]	0.091 [0.05,0.13]	< 0.01 [0,0.004]
Expect. Shock e_t^i	< 0.01 [0,0.004]	< 0.01 [0,0.01]	< 0.01 [0,0.01]	< 0.01 [0,0.006]	< 0.01 [0,0.009]	< 0.01 [0,0.007]	0.120 [0.08,0.17]
$h = 12$							
Cost-Push Shock u_t	0.297 [0.20,0.44]	0.03 [0.01,0.06]	0.021 [0,0.06]	0.133 [0.09,0.20]	0.033 [0.02,0.07]	0.032 [0.02,0.06]	0.022 [0.01,0.06]
Natural Rate Shock r_t^n	0.078 [0.04,0.13]	0.198 [0.12,0.29]	0.072 [0.01,0.14]	0.081 [0.04,0.13]	0.125 [0.07,0.19]	0.158 [0.09,0.24]	0.075 [0.01,0.14]
MP Shock ε_t	0.127 [0.09,0.17]	0.201 [0.12,0.32]	0.666 [0.40,0.96]	0.152 [0.11,0.20]	0.206 [0.11,0.32]	0.202 [0.12,0.31]	0.610 [0.37,0.87]
Expect. Shock e_t^π	0.187 [0.13,0.26]	0.02 [0.01,0.04]	0.02 [0,0.06]	0.323 [0.23,0.43]	0.022 [0.01,0.04]	0.021 [0.01,0.04]	0.018 [0.01,0.06]
Expect. Shock e_t^{y1}	0.288 [0.17,0.40]	0.515 [0.38,0.66]	0.206 [0.02,0.40]	0.288 [0.18,0.39]	0.574 [0.43,0.70]	0.542 [0.41,0.68]	0.210 [0.05,0.39]
Expect. Shock e_t^{y0}	0.014 [0.01,0.03]	0.03 [0.01,0.05]	0.01 [0,0.04]	0.014 [0.01,0.03]	0.03 [0.01,0.05]	0.037 [0.02,0.06]	0.01 [0,0.04]
Expect. Shock e_t^i	< 0.01 [0,0.004]	< 0.01 [0,0.006]	< 0.01 [0,0.003]	< 0.01 [0,0.005]	< 0.01 [0,0.006]	< 0.01 [0,0.005]	0.05 [0.03,0.07]
$h = 20$							
Cost-Push Shock u_t	0.271 [0.18,0.41]	0.03 [0.01,0.06]	0.02 [0,0.05]	0.12 [0.08,0.19]	0.032 [0.01,0.06]	0.031 [0.01,0.06]	0.017 [0,0.04]
Natural Rate Shock r_t^n	0.075 [0.04,0.13]	0.193 [0.11,0.29]	0.07 [0.01,0.13]	0.076 [0.04,0.13]	0.126 [0.07,0.20]	0.155 [0.09,0.24]	0.073 [0.02,0.13]
MP Shock ε_t	0.14 [0.08,0.23]	0.223 [0.12,0.39]	0.53 [0.26,0.90]	0.16 [0.1,0.25]	0.227 [0.12,0.39]	0.226 [0.12,0.39]	0.481 [0.24,0.83]
Expect. Shock e_t^π	0.171 [0.11,0.24]	0.02 [0.01,0.04]	0.015 [0,0.05]	0.286 [0.20,0.40]	0.022 [0.01,0.04]	0.021 [0.01,0.04]	0.015 [0,0.05]
Expect. Shock e_t^{y1}	0.32 [0.20,0.45]	0.499 [0.35,0.64]	0.35 [0.07,0.57]	0.336 [0.22,0.49]	0.554 [0.40,0.68]	0.522 [0.37,0.66]	0.362 [0.11,0.58]
Expect. Shock e_t^{y0}	0.014 [0.01,0.03]	0.03 [0.01,0.05]	0.01 [0,0.04]	0.014 [0.01,0.03]	0.03 [0.01,0.05]	0.036 [0.02,0.06]	0.01 [0,0.04]
Expect. Shock e_t^i	< 0.01 [0,0.003]	< 0.01 [0,0.006]	< 0.01 [0,0.003]	< 0.01 [0,0.005]	< 0.01 [0,0.005]	< 0.01 [0,0.005]	0.037 [0.02,0.06]

Table 3 - Forecast Error Variance Decomposition.

Note: The table reports shares of the variance of inflation, the output gap, the nominal interest rate, expected inflation, expected output gap (one and two-period ahead), and expected nominal interest rate, that are explained by each structural and expectational shock. The entries in the table denote posterior means calculated over the last 10,000 MCMC draws; the numbers below each entry in square brackets denote 95% posterior density intervals. The variance decomposition is calculated for business cycle horizons equal to 4, 12, and 20 quarters.

	u_t	r_t^n	ε_t	e_t^{π}	$e_t^{y_1}$	$e_t^{y_0}$	e_t^{i}
- Learning about trend							
Output Gap	0.014 [0,0.03]	0.113 [0.07,0.16]	0.289 [0.18,0.44]	0.088 [0.03,0.19]	0.457 [0.32,0.61]	0.028 [0.01,0.06]	< 0.01 [0,0.01]
Inflation	0.145 [0.08,0.22]	0.038 [0.02,0.06]	0.082 [0.04,0.14]	0.424 [0.29,0.57]	0.287 [0.16,0.44]	0.017 [0.01,0.04]	< 0.01 [0,0.002]
- Output Gap - CBO's Pot. GDP							
Output Gap	0.033 [0.01,0.07]	0.172 [0.10,0.28]	0.226 [0.10,0.36]	0.016 [0.01,0.03]	0.518 [0.29,0.72]	0.027 [0.01,0.07]	< 0.01 [0,0.004]
Inflation	0.506 [0.40,0.61]	0.013 [0,0.03]	0.147 [0.10,0.21]	0.248 [0.19,0.32]	0.075 [0.02,0.19]	< 0.01 [0,0.01]	< 0.01 [0,0.003]
- Theoretical Output Gap							
Output Gap	0.039 [0,0.09]	0.136 [0.05,0.26]	0.076 [0.01,0.18]	0.028 [0.01,0.09]	0.671 [0.42,0.84]	0.041 [0.01,0.10]	< 0.01 [0,0.002]
Inflation	0.479 [0.07,0.58]	< 0.01 [0,0.02]	0.149 [0.03,0.23]	0.304 [0.12,0.60]	0.049 [0,0.62]	0.01 [0,0.11]	< 0.01 [0,0.003]
- Corr$(u_t, r_t^n) \neq 0$							
Output Gap	0.017 [0.01,0.04]	0.156 [0.08,0.25]	0.328 [0.20,0.40]	0.020 [0.01,0.05]	0.447 [0.31,0.57]	0.021 [0.01,0.04]	< 0.01 [0,0.009]
Inflation	0.172 [0.11,0.27]	0.093 [0.04,0.17]	0.163 [0.10,0.25]	0.138 [0.08,0.23]	0.408 [0.27,0.54]	0.016 [0.01,0.03]	< 0.01 [0,0.005]
- Different Gain Coefficients							
Output Gap	0.039 [0.02,0.07]	0.194 [0.13,0.28]	0.233 [0.13,0.37]	0.028 [0.01,0.05]	0.472 [0.34,0.59]	0.024 [0.01,0.04]	< 0.01 [0,0.008]
Inflation	0.262 [0.19,0.36]	0.080 [0.05,0.12]	0.159 [0.10,0.24]	0.171 [0.12,0.24]	0.307 [0.18,0.42]	0.012 [0.01,0.02]	< 0.01 [0,0.005]
- Alternative Initial Beliefs							
Output Gap	0.013 [0.01,0.02]	0.153 [0.09,0.23]	0.258 [0.17,0.38]	0.025 [0.01,0.06]	0.508 [0.41,0.61]	0.013 [0.01,0.03]	0.024 [0.01,0.04]
Inflation	0.174 [0.12,0.23]	0.083 [0.05,0.13]	0.112 [0.07,0.17]	0.194 [0.13,0.30]	0.408 [0.30,0.55]	0.01 [0,0.02]	0.013 [0.01,0.02]
- MSV Solution as Agents' PLM							
Output Gap	0.105 [0.06,0.16]	0.166 [0.10,0.22]	0.073 [0.04,0.11]	0.074 [0.04,0.12]	0.544 [0.39,0.66]	0.028 [0.01,0.07]	< 0.01 [0,0.007]
Inflation	0.320 [0.24,0.40]	0.053 [0.04,0.07]	0.077 [0.06,0.09]	0.215 [0.15,0.27]	0.313 [0.20,0.45]	0.015 [0.01,0.04]	< 0.01 [0,0.003]
- SPF data on $\widehat{E}_{t-1} i_t$, post-1981							
Output Gap	0.037 [0.01,0.08]	0.192 [0.06,0.30]	0.112 [0.04,0.20]	0.055 [0.02,0.13]	0.542 [0.38,0.82]	0.051 [0.01,0.12]	< 0.01 [0,0.012]
Inflation	0.365 [0.23,0.53]	0.041 [0.02,0.07]	0.016 [0.01,0.04]	0.32 [0.18,0.47]	0.228 [0.08,0.50]	0.020 [0,0.06]	< 0.01 [0,0.002]
- TV MP Rule							
Output Gap	0.035 [0.01,0.08]	0.185 [0.10,0.27]	0.244 [0.15,0.37]	0.028 [0.01,0.05]	0.472 [0.34,0.60]	0.025 [0.01,0.05]	< 0.01 [0,0.007]
Inflation	0.284 [0.20,0.38]	0.071 [0.04,0.12]	0.129 [0.08,0.19]	0.190 [0.13,0.26]	0.304 [0.20,0.43]	0.013 [0.01,0.03]	< 0.01 [0,0.005]

Table 4 - Variance Decomposition, robustness to different specifications. *Note:* The table reports the share of variance of output gap and inflation explained by each structural and expectational shock in each of the estimated models (the table refers to an horizon of 20 quarters). The entries denote posterior means calculated over the last 10,000 MCMC draws, along with 95% posterior density intervals (numbers below in brackets).

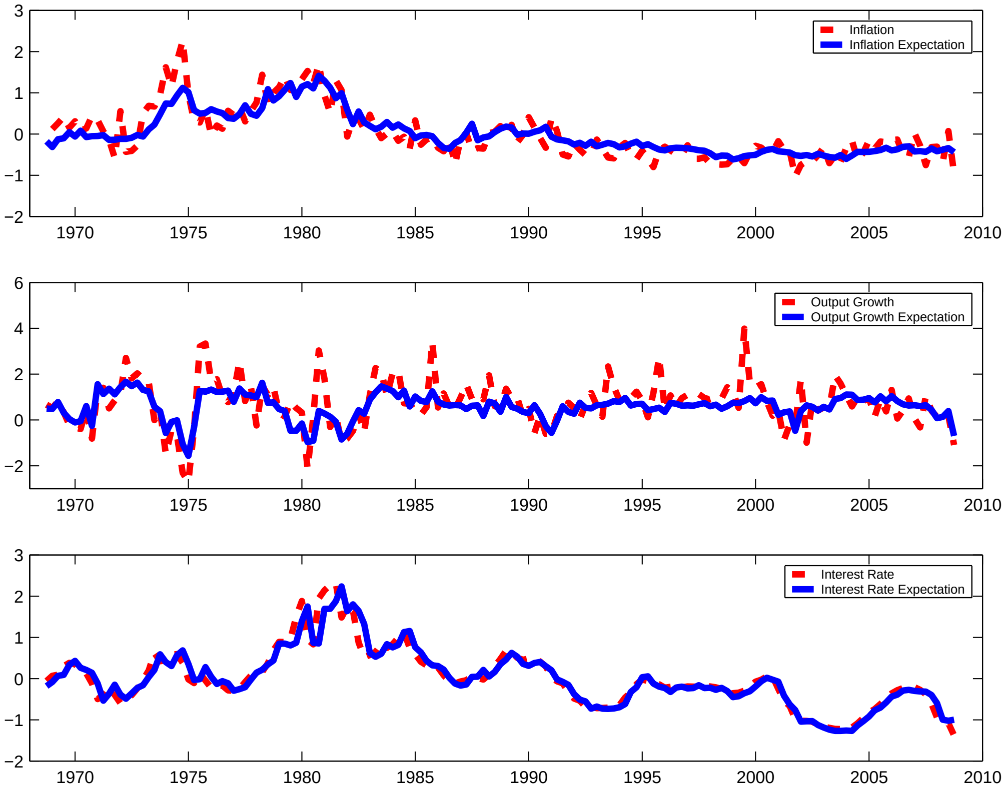

FIGURE 1. Realized Variables and Expectations.

Note: The first panel shows realized inflation (π_{t+1}) and inflation expectations ($\widehat{E}_{t-1}\pi_{t+1}$) from the Survey of Professional Forecasters. The second panel shows output growth (y_t) along with output growth expectations ($\widehat{E}_{t-1}y_t$) from the Survey of Professional Forecasters. The third panel shows the three-month nominal interest rate (i_t) along with interest rate expectations ($\widehat{E}_{t-1}i_t$) extracted from the term structure of interest rates. Realized values and expectations regarding inflation and interest rates are shown in deviation from their sample averages and expressed as quarterly rates.

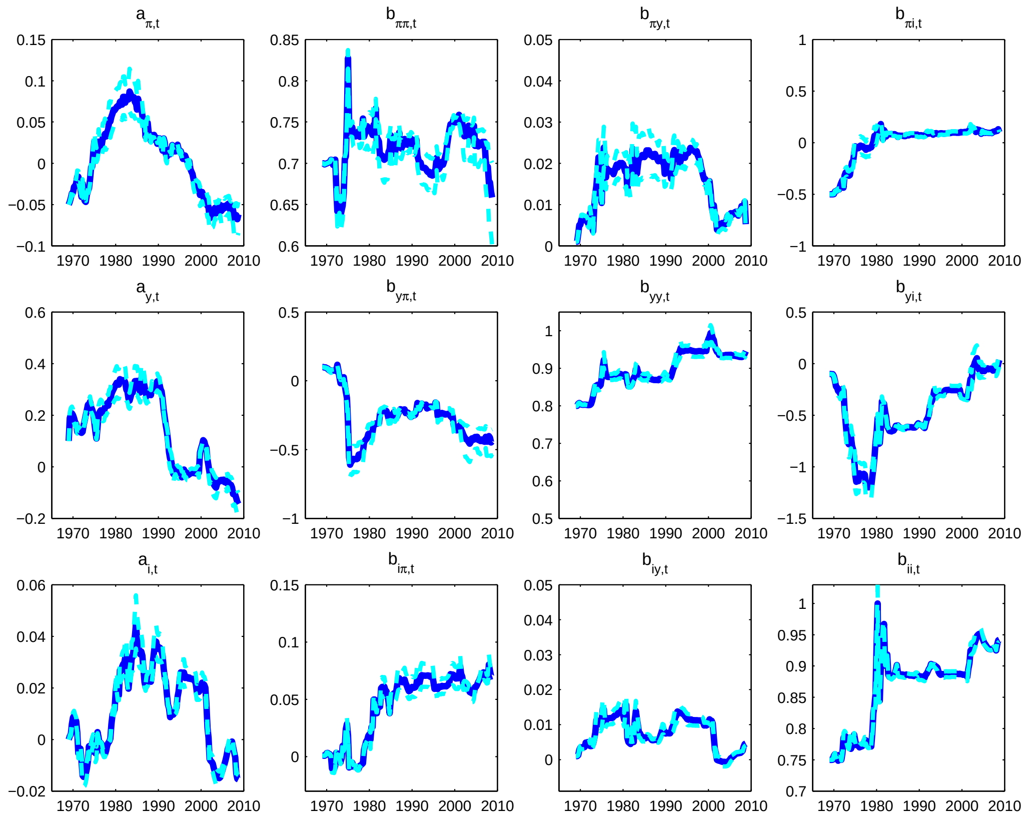

FIGURE 2. Evolution of estimated beliefs over the sample.

Note: The first row shows beliefs related to the perceived law of motion for inflation, the second row beliefs related to the output gap equation, and the third row related to the interest rate equation. Each panel shows the beliefs obtained as averages across Metropolis-Hastings draws, along with 2.5 and 97.5% percentiles. The sample is 1968:III-2009:I.

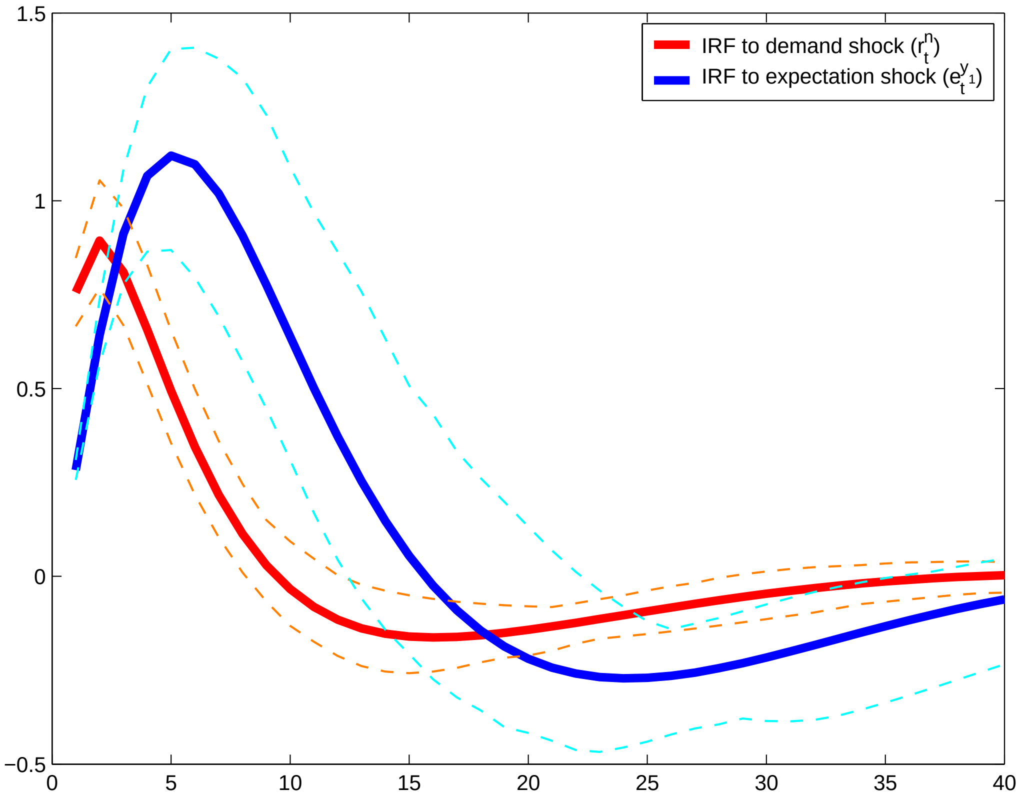

FIGURE 3. Impulse response function of the output gap to the natural rate shock and the expectation shock about future output.

Note: Solids lines in the figure denote mean impulse responses over the sample, calculated over the last 10,000 MCMC draws. Dashed lines denote 95% error bands.

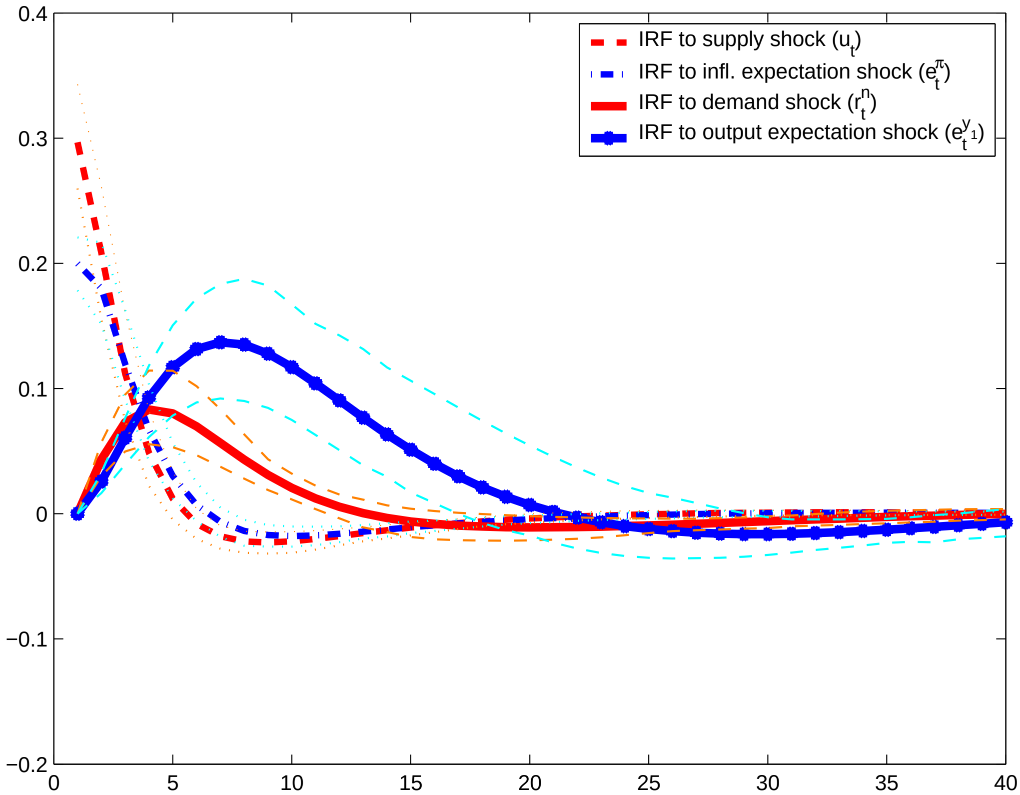

FIGURE 4. Impulse response function of inflation to the cost push-shock, to the natural rate shock, and to the expectation shocks about future output and about future inflation.

Note: Solid lines in the figure denote mean impulse responses over the sample, calculated over the last 10,000 MCMC draws. Dashed lines denote 95% error bands.

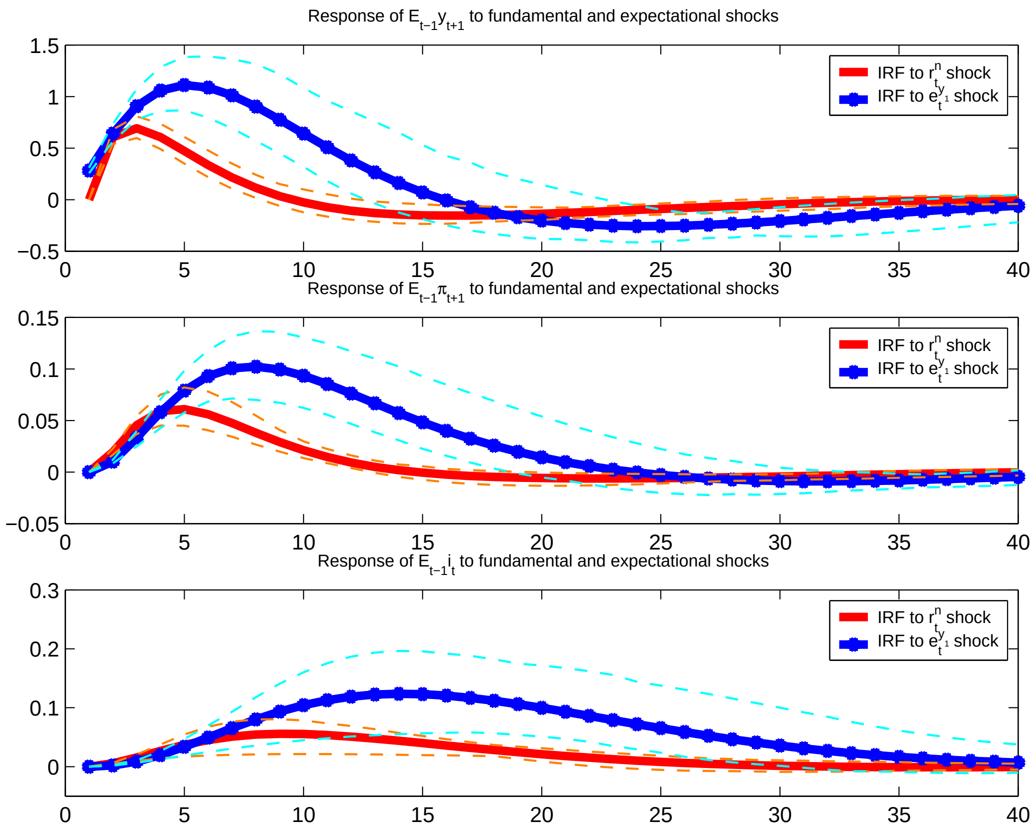

FIGURE 5. Impulse response function of observed output, inflation, and interest rate expectations to structural and expectation shocks.

Note: Solid lines in the figure denote mean impulse responses over the sample, calculated over the last 10,000 MCMC draws. Dashed lines denote 95% error bands.

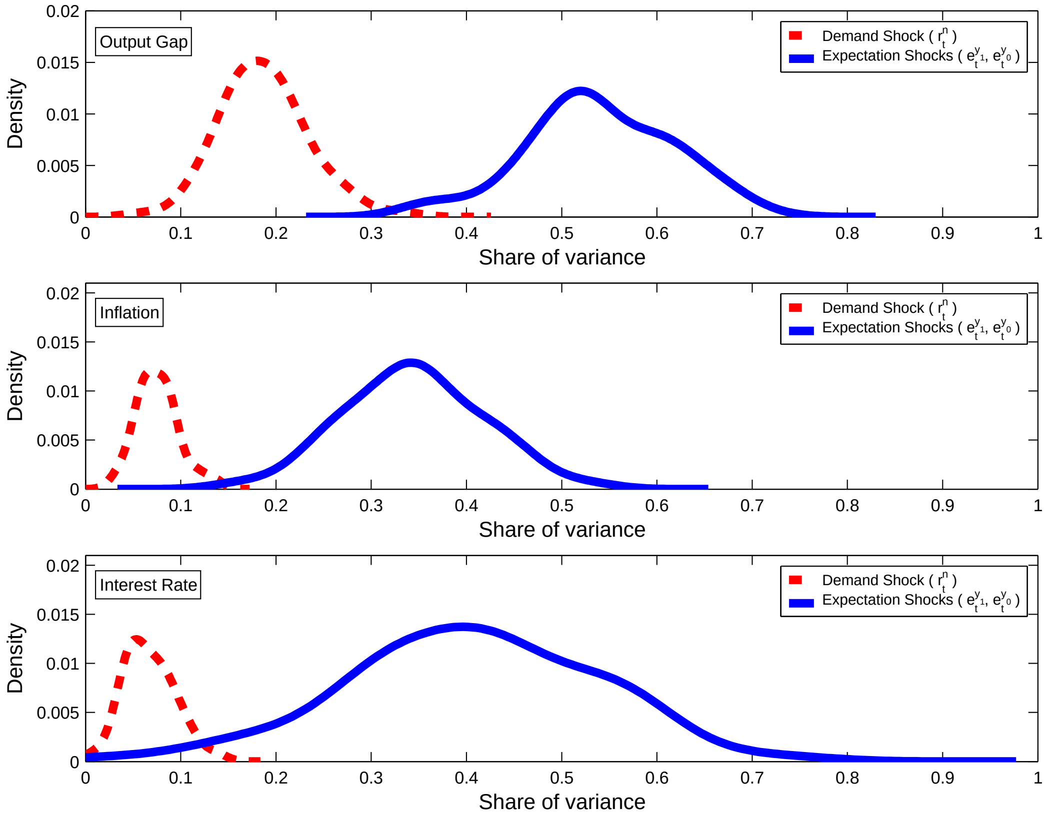

FIGURE 6. Forecast Error Variance Decomposition: posterior densities.

Note: The figure shows the posterior densities for the shares of the variances of output gap (first panel), inflation (second panel), and interest rates (third panel), that are due to the natural rate shock and to the expectation shocks about future output (summing the shares due to $e_t^{y_1}$ and $e_t^{y_0}$). The densities are calculated over the last 10,000 MCMC draws.

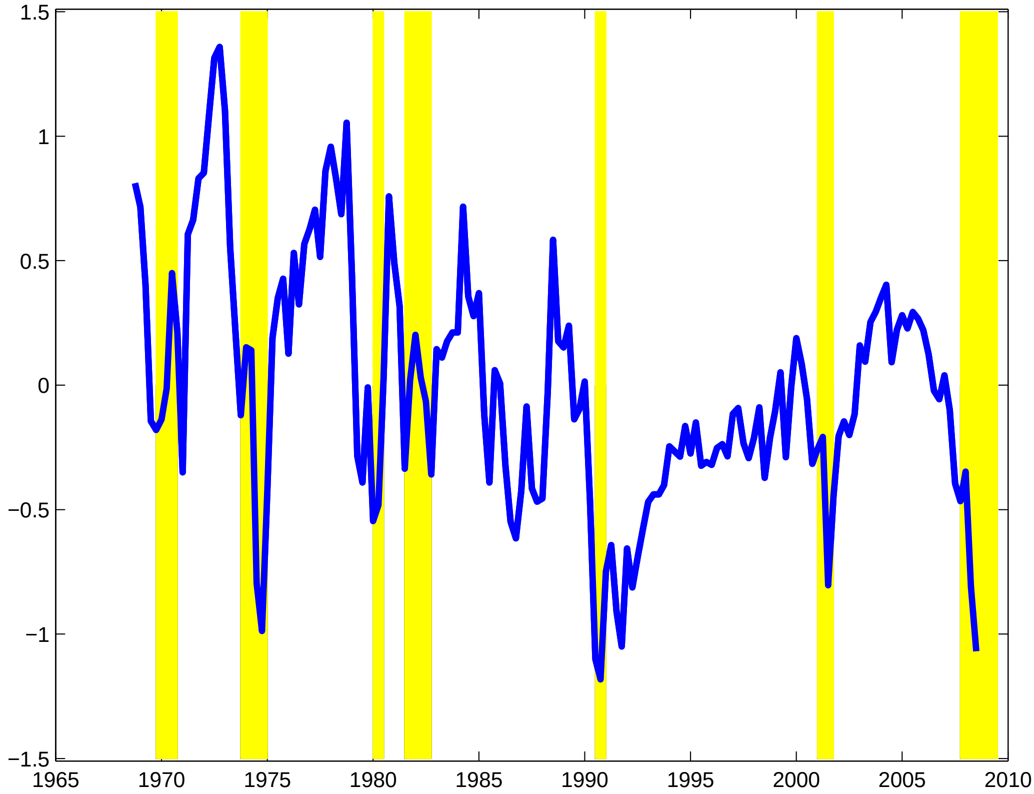

FIGURE 7. Expectation shock about future real activity ($e_t^{y_1}$) and NBER recession dates (yellow vertical bands).

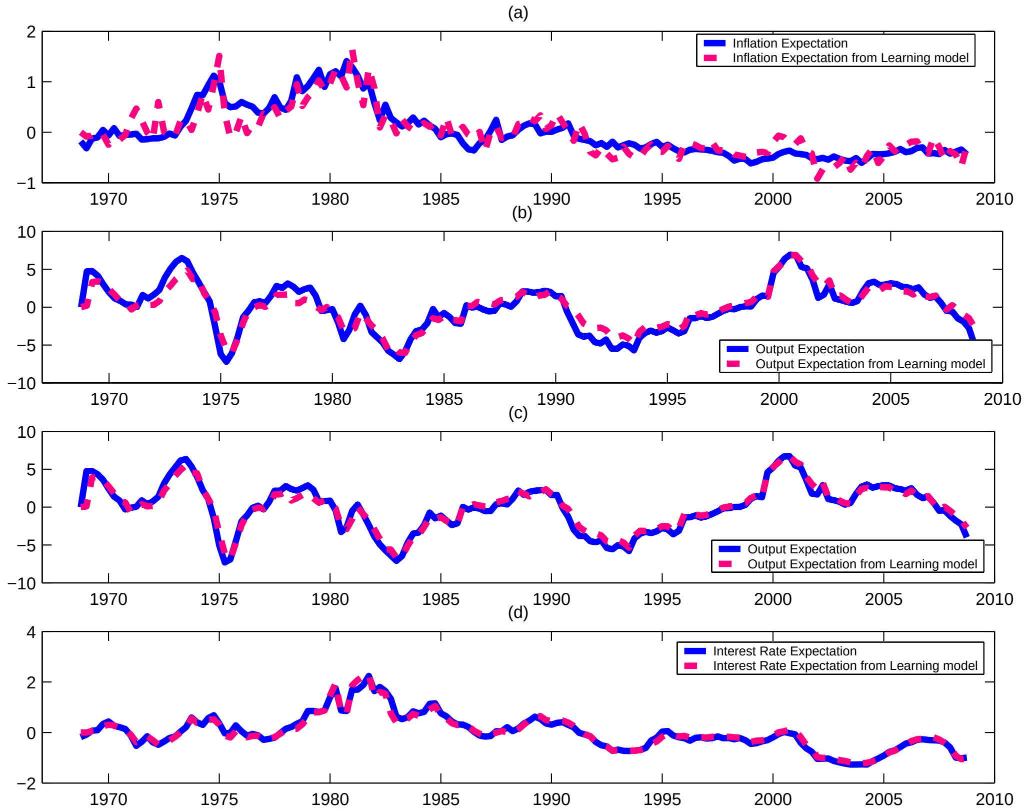

FIGURE 8. Observed Survey Expectations and Expectations from economic agents' PLM.

Note: The first panel compares inflation expectations ($\widehat{E}_{t-1}\pi_{t+1}$) from the Survey of Professional Forecasters (solid line) and the near-rational expectations from the agents' learning model (dashed line), obtained as averages across MCMC draws. The second and third panels show output gap expectations ($\widehat{E}_{t-1}y_{t+1}$ and $\widehat{E}_{t-1}y_t$) from the Survey of Professional Forecasters and near-rational expectations from the learning model. The third panel shows interest rate expectations ($\widehat{E}_{t-1}i_t$) extracted from the term structure of interest rates along with expectations from the learning model.