



**Discussion Papers in Economics**

**CONTAGION OR FLIGHT-TO-QUALITY PHENOMENA IN  
STOCK AND BOND RETURNS**

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# Contagion or Flight-to-Quality Phenomena in Stock and Bond Returns

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## **Abstract**

In this paper, I study the correlation between stock and bond returns. We can define flight-to-quality from stocks to bonds as the decrease in the correlation between the two assets in falling stock markets periods (bear state), since the two assets returns move in the opposite direction. On the contrary, a movement in the same direction between the two asset classes as the economy is at a bear state, can be classified as contagion. Firstly, I show that a two-state model, with regimes characterised as bear and bull states, is required in order to capture and explain the dynamics of equity returns at the bivariate level. Secondly, the analysis I have conducted shows statistically significant evidence of flight-to-quality phenomena from stock to bond returns, in the US and UK for the period 1986-2010. Finally, I have found evidence of time-variation in the structure of the predictability patterns linking financial markets and monetary policy, as the latter is expressed through short-term interest rates. These results have not only important implications for portfolio diversification and asset allocation, but they are also adding to the ongoing debate on how the time variation in the stock-bond correlation is driven by changing macroeconomic conditions.

JEL classification: C32, C58, E44, G1

Keywords: Contagion, Markov-switching, Non-linearities, Regime Changes, Stock-Bond Correlation

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# 1 INTRODUCTION

This paper studies and analyses the relationship between stock and bond markets. Generations of researchers in economics and finance have been involved in research projects towards the understanding of the mechanisms that link these two markets. The understanding of the comovements and correlations between financial assets signifies a key question, which has spawned numerous studies. This interest is motivated by the theoretical importance of stock-bond correlation for price formation and the practical applications for asset allocation, risk management and portfolio diversification.

Stocks and bonds have very different risk-return characteristics. Stocks are expected to yield higher returns than bonds over the long run, even though stocks are more volatile than bonds. As the modern portfolio and diversification theory suggests, by mixing stocks and bonds in a portfolio, investors can achieve the desired level of risk, which depends not only on the risks of individual assets, but also on the comovements of the individual assets in the portfolio. If, for example, we have a portfolio with two assets and their prices tend to move in opposite directions, investing in these two assets would be less risky than investing in each one individually, as the increase/decrease in one asset's price will neutralise the decrease/increase in the other asset's price.<sup>1</sup>

It is well known in the literature that financial time series always undergo episodes in which the behavior of the series seems to change quite dramatically. Such phenomena refer to regime shifts or structural breaks to the parameters of the return-generating process, and usually occur because of economic and financial crises which happen around the world. Some examples of such crises are the Great Crash of 1929, which was a major distribution to the stability and functioning of the principal industrial nations of the world; the oil crisis of 1973, where an increase in oil prices led to economic recession across the US, Europe, Japan and the Third World; and the historic one-day plunge of US stock market in 1987, which spread all over the world. In recent years, we have the currency crises in Mexico (1994), East Asia (1997), Russia (1998), Brazil (1999), Argentina (2002), where speculative attacks on their currencies led to a massive currency devaluation. The late most recent examples are the credit-crunch of 2007-2008 and the sovereign debt crisis of 2010.

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<sup>1</sup>An investor can reduce portfolio risk simply by holding combinations of instruments which are not perfectly positively correlated. In other words, investors can reduce their exposure to individual asset risk by holding a diversified portfolio of assets. Diversification may allow for the same portfolio to bring expected return with reduced risk.

In order to provide intuition for the nature of the time-variation that affects the stock-bond correlation and for the detection of flight-to-quality or contagion phenomena, I will begin with describing the regime switching properties of only stock returns, and then I will proceed with the description of the regime switching properties of stock and bond combined. For this reason, starting from a simple univariate model of stocks, I will then move to the bivariate-joint distribution of the two assets in the US and UK for the period 1986-2010.

Having estimated a relatively large range of models I have found evidence that models in which besides the conditional mean, variance and covariance are also state-dependent and therefore change over time, are strongly required by the data in order to provide a good fit.<sup>2</sup> The analysis suggests that a simple two-state (bull-bear) model is able to capture the time-varying volatility in stock and bond return series. More precisely, and opposite to Guidolin and Timmermann (2005 and 2006), who they found either three (UK) or four-state (US) models respectively, I identify two states that can broadly be interpreted as a high volatility bear state with negative mean returns and a highly persistent bull state with positive mean return and low volatility.

The difference between my results and these two studies arises from the fact that I am using a different dataset. Guidolin and Timmermann (2005) uses monthly returns on the FTSE All stock market index and on a 15-year government bond for the period 1976 to 2000 (300 observations), while in Guidolin and Timmermann (2006) they use returns on all the common stocks listed on the NYSE (small and large stock returns) and on a 10-year government bond for the period 1954-1999 (522 observations). On the other hand, my data consists of total stock return index and total bond return index for the period between 1984-2010, a total of 298 monthly observations. Those 10 years from 2000 onwards are usually described as the era of "New Economy" and economic globalisation. The most significant change in modern economics is the unprecedented increase in the size and integration of financial markets and financial institutions across the world. This resulted financialisation<sup>3</sup> led to an increase of the interlinks between markets and between economies. During the last decade there has been an increase in economic volatility, in the number of economic crises and consequently in economic uncertainty, which has been the direct result of financial development.

Regarding the monthly volatility between stock and bond returns, I found that for the bivariate

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<sup>2</sup>I define good fit in terms of in-sample analysis, using information criteria and linearity tests.

<sup>3</sup>Generally, we can define "financialisation" as the increasing size and integration of capital markets (equity, bonds, derivatives, foreign exchange) and financial intermediaries (banks, investment funds, insurance, pensions) within and between countries.

US and UK models the correlation is positive during expansion periods (bull states) and becomes negative in recession periods (bear states). This negative correlation in bear states is consistent with many previous studies (Ilmanen (2003), Connolly, Stivers and Sun (2005)) and indicates flight-to-quality phenomenon as investors sell what they perceive to be high-risk investments and move towards safer markets, such as the bond market, in falling stock markets times.

Furthermore, on top of the bivariate case, I am also dealing with the linkages between financial returns and macroeconomic variables (see Fama (1981), Bredin, Hyde, Nitzsche and O'Reilly (2007)). The literature has identified a number of channels through which monetary policy might have contributed to the build-up in financial imbalances. One macroeconomic variable that may affect the stock-bond return correlation is short-term interest rate (3-month Treasury bill). As Merrouch and Nier (2010) argued, most of these channels are thought to have worked through policy rates that were kept low for too long, especially after 2000.<sup>4</sup> In theory, we know that interest rate increases go hand in hand with falling stock markets, whereas rate cuts boost stocks. This is actually the rule of thumb driving the stock-bond correlation during bull states (see Figures 1 and 3). This rule has, however, been broken in the last 10 years, resulting to a negative stock-bond correlation.

What is more, two factors justify the decision of using a 3-month horizon. First, the horizon has to be long enough, so that interest rates reflect investors expectations about future economic situation. In other words, interest rates embodied the forward-looking trend of investors' preferences. At the same time, and according to Favero and Giavazzi (2002): "...the horizon should not be too long, otherwise spreads would average expectations over long periods of time, and would thus fail to capture the expectation of a crisis precisely enough...". Based on that, and in order to examine the effects of monetary policy on stock-bond correlation, I have estimated threevariate models consisting of stock, bond and interest rate returns. Results are similar to those from the bivariate case, and flight-to-quality phenomena continue to exist. By this I mean that a crash in stock market, which is captured by a bear state, is accompanied by a boom in the government bond market. So, the two markets move into different directions and as a result the correlation between them is decreasing in falling stock markets.

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<sup>4</sup>A study by De Nicolo, Dell'Ariccia, Laeven and Valencia (2010) show how ambiguous the effect of low short-term interest rates could be on risk-taking. As argued by some economists, the US Federal Reserve had kept interest rates low for too long after the 2001 recession. Others argue that policy rates had been unusually low globally ahead of the 2007-2008 financial crisis.

The evidence that I found of time-variation in the structure of the predictability patterns linking financial markets and the economy means that even though expected returns and economic conditions (as captured by interest rate) have been subject to recurring states, the dynamic linkages between financial prices and monetary variables are unstable and vary over time. To be more precise, bond returns become highly predictable in both states using lagged values of interest rates (which forecast statistically significant lower returns since bond prices move inversely to interest rates). Stock returns are also predictable by past interest rate growth, although the economic effect is rather negligible.

The statistical tool that I will use for the purposes of this study is the Markov-Switching (MS) model, with which I will test for discontinuities in the data-generating process of the return series under investigation. With MS models the mean (expected returns), the variance (conditional volatility) and the variance-covariance matrix can take on different values depending on the realisation of the latent state variable  $S_t$ , which is assumed to follow a Markov process and takes values between 1 and  $k$  - where  $k$  is the number of states-regimes. Likewise, the transition between different regimes is governed by a transition probability matrix, and by constructing the regime probability - the probability that tomorrow's regime is the first regime given current and past information - we are able to know in what regime the economy is at each point. In other words, a crisis can be thought of as a switch from a state of the world with low and/or negative stock return and high volatility (a bear state) to one where the mean returns are high and/or positive and the volatility very low (a bull state); and vice versa. With this particular class of models we are testing for jumps in the mean and changes in the volatility of the time series across different regimes. Additionally, by using the MS approach we let the data describe the features of the different phases of the economy, and most importantly there is no arbitrary selection of crisis and non-crisis periods, since these are endogenously determined.<sup>5</sup>

The remainder of the paper is organised as follows. Section 2 provides a literature review, while Section 3 analytically studies the econometric representation which I use to give empirical content to the theory. In other words, it describes the Markov-Switching (MS) model and its estimation.

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<sup>5</sup>The choice of both crisis and non-crisis periods is very important and reflects a particularly difficult problem in the financial contagion literature. A great number of studies - Glick and Rose (1999), Van Rijckeghem and Weder (2001), Forbes and Rigobon (2002) and Dungey, Fry, Gonzalez-Hermosillo and Martin (2002) - are based on an ad hoc selection (newspaper or personal view) in order to determine the dating of crises. On the contrary, the MS model demonstrates a more objective procedure for dating crises based on the data characteristics.

Section 4 describes the data, while Section 5 provides a range of results from specification tests. Section 6 reports the empirical results and interprets the findings. Finally, Section 7 contains the concluding remarks.

## 2 LITERATURE REVIEW

### 2.1 Stock-Bond Correlation

The relationship between stock and bond returns has received considerable attention in literature, where there is evidence of either flight-to-quality or contagion phenomena. For instance, in bear market periods, investors shift their preferences from stock to bond market, resulting to some divergence in the returns between these two asset classes. This is the so-called flight-to-quality effect, which implies that periods of negative stock-bond correlation coincide with stock market crashes - as the economy moves from a bull to bear state, the correlation from positive becomes negative. In the case of the opposite, falling stock markets considerably increase the correlation between the two assets, contagion phenomena may be arise.

The stock-bond correlation has been a fundamental input to the management and diversification of a portfolio. The lower the correlation between the two assets, the greater their diversification potential and the more attractive a combination of the two becomes. In the literature, there are either studies which impose a constant relationship between stock and bond returns, where the correlation is time invariant (Campbell and Ammer (1993)), or studies (Scruggs and Glabadanidis (2003)) which strongly reject the constant correlation restriction on the covariance matrix between stock and bond returns. However, general evidence has shown that the correlation varies over time and under exogenous influences. Or, to put it differently, it changes as the economy moves from a bear to bull state. These asymmetric characteristics of the correlation are well captured by regime-state switching models as Ang and Bekaert (2002) argued and as I will explain later on in this paper.

One of the first who contended that the stock and bond comovement is state-dependent was Barsky (1989), who also concluded that the two assets move in opposite directions during bear periods. Furthermore, this comovement may vary with stock market uncertainty, as suggested by David and Veronesi (2000). More precisely, according to Connolly *et al.* (2005) and Stivers and



Sun (2002), US stock and bond returns tend to move together (positive correlation), at significant extent, during periods of lower stock market uncertainty (low-volatility bull state), while they move in the opposite direction (negative correlation), continuously, during periods in which stock market uncertainty is high (high-volatility bear state). This comovement between the two assets is consistent with the flight-to-quality argument which happens when the correlation between stocks and bonds strongly decreases in falling stock markets, since the two assets move in opposite directions. Another study by Ilmanen (2003) has shown how differently the two assets react to growth. During bull times, stocks tend to outperform bonds, and during bear times bonds outperform stocks. On the other hand, when the two asset classes move in the same direction in falling stock markets (bear states), which implies that the correlation is higher than the one during bull states, there is evidence of contagion.

By contrast, there is a paucity of studies which detect contagion phenomena between stock and bond returns. One of these, by Jensen and Mercer (2003), documents that the monthly correlation for the two US assets for the period 1972-1999 is lower during expansions than during recessions. There is a marked increase in the correlation during turmoil periods, which is statistically significant for small cap stocks only, but not for large cap stocks. The higher correlation, which is inconsistent with the flight-to-quality, has an important implication for the portfolio allocation, implying that the investor loses some of the diversification benefit during recessionary periods and increases the risk.

## **2.2 Identifying Regime Shifts**

There is a widespread disagreement in the existing literature about what contagion is. Some economists believe that a crisis that starts from one economy and spreads on to another - when the two economies are located in separate geographic regions, with different structures and weak cross-market linkages - is contagion. Others prefer to use the term shift-contagion. According to this term, contagion is the significant increase in cross-market linkages after a shock to one country or to a group of countries. However, in the case of two countries having a high degree of comovement before the shock and continuing of being highly correlated after the crisis, one may suggest that this does not constitute contagion. The World Bank provides three definitions of contagion (broad, restrictive and very restrictive), while according to Pericoli and Sbracia (2003) there are five. In

this paper I shall use Forbes and Rigobon (2002) definition, where contagion is the significant increase in correlation/comovement between countries/markets, conditional on a crisis occurring in one country/market or a group of countries/markets. The aim of this correlation-based definition is to try to identify whether a shock to the returns of one asset has a different impact on the level of returns in another market during a financial crisis compared to a non-crisis period.

In order to detect and identify significant changes in the distribution of the two asset returns, many different approaches have been proposed in the literature. We can classify them into two classes. In the first class, we have empirical analyses that simply attempt to measure the effect of a shock in one country/market on another country/market. These studies employ the threshold principle and they use probit/logit models where the initial shock is an extreme value of an indicator of speculative pressures (Eichengreen, Rose and Wyplosz (1996), Forbes (2001) and Van Rijckeghem and Weber (2001)); leading indicator approaches, where there is a parsimonious set of indices of vulnerability to external or internal shocks in order to forecast crises (Kaminsky, Lizondo and Reinhart (1998) and Berg and Pattillo (1999)), and finally, volatility-based studies using GARCH models (Engle, Ito and Lin (1990) and Hamao, Masulis and Ng (1990)), which deal with the transmission of volatility shocks and, like the two previous studies, they do not assume any kind of structural break during the crisis.

In recent years, Ang and Bekaert (2002) showed that GARCH type models are unable to take account of the higher correlations that stock markets face during bear states as opposed to correlations during bull states. Not accounting for these structural shifts in the volatility process causes GARCH models to overestimate the persistence of volatility. Hence, it is the regime or the nonlinearity that is important rather than the changing volatility. What is more, studies like the one conducted by Turner, Startz and Nelson (1989), Perez-Quiros and Timmermann (2000), Guidolin and Timmermann (2003 and 2005) have shown that regime switching models that account for different phases in the business cycle are quite successful in this regard. Therefore, in the second group of empirical works, we test the discontinuities in the data-generating process and we have: 1) tests of structural breaks in the correlation coefficient (Corsetti, Pericoli and Sbracia (2001), Forbes and Rigobon (2002) and Rigobon (2003)), and 2) Markov-Switching models, which directly test the presence of multiple equilibria (Jeanne and Masson (2000) and Fratzscher (2003)).<sup>6</sup>

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<sup>6</sup>For a complete review of test of contagion see Dornbusch, Park and Claessens (2000) and Pericoli and Sbracia (2003).

In this paper, I will concentrate in this latter strand of empirical literature and especially in Markov switching models which were built by Hamilton (1989) and allow the data to be drawn from two or more possible regimes (distributions). The transition from one regime to another is driven by the realisation of a discrete variable (the regime-state), which follows a Markov chain process. In other words, this class of models imposes that in each point of time there is a certain probability that the process will stay in the same regime in the next period or it might transition to another regime in the next period.

In the presence of regime switching dynamics between return assets, Guidolin and Timmermann (2006), having explored a variety of econometric models for the joint distribution of US stock and bond returns for the period 1954-1999, found that a four-regime model is considered most appropriate with regimes characterised as crash, low growth, bull and recovery. Furthermore, they detected flight-to-quality phenomena, since the negative correlation between the large cap stocks and the 10-year T-bonds in the crash-bear state (-0.40) indicates that there are outflows of capital from the stock market to the bond market when stocks are in turmoil. In another study (Guidolin and Timmermann, 2005), this time on the UK stock and bond returns (1976-2000), they identified that a three-state Markov-switching vector autoregressive (MS-VAR) model with states interpreted as bull, normal and bear is required to capture the time-variation in the mean, variance and correlation between those two assets. Moreover, the correlation between the stock and the 15-year government bond returns varies substantially across regimes, switching from 0.55 in the bull state to -0.45 in the bear state. The two asset returns do not move closely together in the bear state, while correlations are positive and significant in the normal and bull state. Even when these studies extend the joint distribution, by allowing for predictability from one predictor variable (dividend yield), flight-to-quality effects continue to exist (correlations have the same sign across the regimes), but now the magnitude is less than before.

### **2.3 Monetary Policy and Stock-Bond Correlation**

Surprisingly, little is known about the driving forces behind the time-varying correlation between stock and bond returns (e.g. Li (2002), Baele, Bekaert and Inghelbrecht (2010)). For that reason, it is natural to ask whether it is sensible to assume that such dynamic predictability relationships (if any) have been stable or not over time. This question arises from the fact that the US and the

UK economy have been changing at a fast pace over the last 30 years. What is more, stock and bond markets have been subject to dramatic changes that may lend support to the hypothesis of unstable dynamic linkages. If we understand primarily the links between monetary policy and asset prices, then we will be a step closer to understanding the policy transmission mechanism.

In this paper, I am also extending the joint distribution (bivariate model) of stocks and bonds by including one additional predictor variable, the short-term interest rate (trivariate model). The effect of monetary policy, as expressed by interest rates, on the correlation of stock and bond returns, is of great importance, not only to macroeconomists but also to financial economists. It is well mentioned in the literature, e.g. Bernanke and Kuttner (2005), that monetary policy shocks are transmitted to equity valuations through a number of alternative channels, for example changes in the cost of capital, changes on aggregate demand through changes in equity prices (wealth effect), changes in the values and composition of optimal private portfolios or by other mechanisms as well. In their study, the authors also present evidence that stock returns respond to monetary policy shocks. While many predictor variables have been proposed (such as the default spread, the dividend yield, the inflation or even the aggregate consumption and others) one of the key instruments that have received less attention is the interest rate. Much of the transmission of monetary policy comes, according to Rigobon and Sack (2004), from the influence of interest rates (and especially short-term interest rates, as they argued) on other asset prices.

In addition, this approach is directly relevant to the large literature in finance that has reported evidence of predictability in stock and bond returns. From one side, we have studies exploring the linear analysis of the relationship between stock markets and the macroeconomy. For example, Fama (1981) and Canova and De Nicolo (2000), using ordinary least squares (OLS) and vector autoregression (VAR) models respectively, investigated the links between stock returns, interest rates, inflation and real activity. Others, like Guidolin and Timmermann (2005 and 2006), Guidolin and Ono (2006) and Guidolin and Hyde (2010) stressed that asset returns contain predictability patterns that can not be described by simple linear models. For that reason, these latter studies take account of both possible regime switching and macroeconomic influences on US and UK stock and bond returns using a markov switching approach in order to accommodate for non-linearity.<sup>7</sup> In

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<sup>7</sup>Guidolin and Ono (2006) and Guidolin and Hyde (2010) estimate "heavier" models using three asset returns (stock return, 10-year Government bond and 1-month Treasury bill) and a large set of predictive variables (CPI inflation, industrial production, dividend yield and unemployment rate among others).

this paper, the specification tests that I have performed show that stock and bond returns become highly predictable using lagged (past) values of interest rate. In other words, I provide evidence that the best fit to the threevariate specification is given by a two-state model in which the vector autoregressive components are regime-dependent. This means that the dynamic linkages between equity markets and the monetary policy have been unstable over time. Next session presents the econometric framework that I have been using in order to achieve my goal.

### 3 MODEL

In recent years, the interest of economists has turned to the modelling of non-linearities in econometric time series. This is because phenomena such as regime shifts in financial and macroeconomics time-series cannot be modelled implicitly using linear time-series model, in the tradition of Box and Jenkins (1970). The time-series modelling of regime shifts began when Quandt (1958) introduced the switching regression model. Then, Goldfeld and Quandt (1973) extended the switching regression model to allow the regime shifts to follow a Markov chain, where the regime shift is serially dependent. They called it the Markov switching regression model. Based on Goldfeld and Quandt's ideas, Hamilton (1989) tried to characterise changes in the parameters of an autoregressive process. In order to measure macroeconomic fluctuations on the U.S. business cycle, he employed a Markov Switching Autoregressive (MS-AR) time series model, where, as the economy may either be in a fast growth or slow growth phase, the switch between the two states was governed by the outcome of a Markov process.

MS models have generally been adopted in the literature by researchers who were interested in describing and explaining some specific features of economic time series, such as the volatility clustering (Pagan and Schwert (1990)), the business cycle asymmetries (Hamilton (1989) and Clements and Krolzig (1998)), the non-linear dynamics of asset returns (Guidolin and Timmermann (2005 and 2006)), or the implications of the return predictability into portfolio diversification (Guidolin and Hyde (2010)). Dungey, Fry, Gonzalez-Hermosillo and Martin (2005) tried to unify all the different empirical approaches on the existence of contagion - the correlation analysis of Forbes and Rigobon (2002) based on crisis and non-crisis periods, the probability-based model of Eichengreen *et al.* (1995), the Favero and Giavazzi (2002) VAR approach based on modeling increases in volatil-

ity, and the latent factor model approach by Corsetti *et al.* (2001) - by showing how each of these methods is nested within a latent factor framework similar to that of Corsetti *et al.* (2001). In other words, they showed that many of the tests of contagion can be viewed as tests of structural breaks.

An alternative approach, the one that I follow, involves specifying a Markov switching model. The Markov switching model assumes that the parameters of the underlying data-generating process of the observed time series  $r_t$  depend upon the unobservable regime variable  $S_t$ . This process implies that  $r_t$  depends only on the most recent value  $r_{t-j}$  - where  $j$  the number of the autoregressive components. In other words, the movements between regimes or regime shifts enable probabilistic statements to be made regarding the likelihood of the series being in a particular regime at any particular time. Regime shifts can happen exogenously and the probability of different regimes is called the transition probability. The latter identifies which regimes occur at each point in time, rather than imposing particular dates a priori. Consequently, it allows the data to reveal the nature and frequency of significant shifts.

### 3.1 Univariate Markov Switching Model

The first step is to assess the presence of regimes in the individual stock return series and to consider the degree of coherence across the state variables characterising the regimes in the returns of stock markets. For that reason, first I will entertain a zero lag autoregressive process, AR(0), which satisfies the following equation:

$$y_t = \mu + \varepsilon_t \tag{1}$$

where  $y_t$  is the return of an asset at time  $t$ ,  $\mu$  is the intercept and  $\varepsilon_t$  is an independently identically distributed (*iid*) random variable of innovations whose elements have zero mean ( $E(\varepsilon_t) = 0$ ) and variance  $\sigma^2$  ( $E(\varepsilon_t^2) = \sigma^2$ ),  $\varepsilon_t \sim iid(0, \sigma^2)$ . Introducing one autoregressive component, AR(1), Equation 1 will become:

$$y_t = \mu + \phi_1 y_{t-1} + \varepsilon_t \tag{2}$$

where  $\phi$  is the coefficient of lag 1 and  $y_{t-1}$  is the return of the asset at time  $t - 1$ . For the process to be stationary we require that the parameter  $\phi$  satisfies the restriction that  $|\phi| < 1$ , which means that there is a covariance-stationary process for  $y_t$  satisfying Equation 2. In more general form, for

an AR( $p$ ) process we have:

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t = \mu + \sum_{i=1}^p \phi_i y_{t-i} + \varepsilon_t \quad (3)$$

The basic Markov switching model can be described as a generalisation of Equation 1:

$$y_t = \mu_{S_t} + \varepsilon_t \quad (4)$$

where now  $S_t = 1, 2, \dots, k$  denotes the unobserved state indicator which follows an ergodic  $k$ -state Markov process with finite number of states and state dependent intercepts  $\mu_{S_t}$ . The next step is to introduce state-independent autoregressive components:

$$y_t = \mu_{S_t} + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t = \mu_{S_t} + \sum_{i=1}^p \phi_i y_{t-i} + \varepsilon_t \quad (5)$$

and by allowing them to be state-specific we get:

$$y_t = \mu_{S_t} + \phi_{1,S_t} y_{t-1} + \phi_{2,S_t} y_{t-2} + \dots + \phi_{p,S_t} y_{t-p} + \varepsilon_t = \mu_{S_t} + \sum_{i=1}^p \phi_{i,S_t} y_{t-i} + \varepsilon_t \quad (6)$$

Furthermore, we can let the return innovation term to have state-specific variance  $\sigma_{s_t}^2$  and to be normally distributed conditional on  $S_t = s_t$ .

According to Hamilton (1994), the variable  $S_t$  is assumed to follow a first order, homogenous Markov process. This implies that the current regime-state  $s_t$  only depends on the regime one period ago,  $s_{t-1}$ . Hence, the model is completed by defining the transition probabilities of moving from one state to the other and the transition probability matrix  $\mathbf{P}$  is given by:

$$\mathbf{P}_{[i,j]} = \Pr \{S_t = j \mid S_{t-1} = i\} = p_{ij}, \quad i, j = 1, \dots, k \quad (7)$$

or in matrix formation:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{k1} \\ p_{12} & p_{22} & \cdots & p_{k2} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1k} & p_{2k} & \cdots & p_{kk} \end{bmatrix} \quad (8)$$

where  $p_{ik} = 1 - p_{i1} - p_{i2} - \dots - p_{i,k-1}$  for  $i = 1, \dots, k$ . Also, all the elements of the transition matrix  $\mathbf{P}$  must satisfy the following condition:  $\sum_{j=1}^k p_{ij} = 1, \forall i, j \in \{1, \dots, k\}$  or  $p_{i1} + p_{i2} + \dots + p_{ik} = 1$ . Thus,  $p_{ij}$  is equal to the probability that the Markov chain moves from state  $i$  at time  $t-1$  to state  $j$  at time  $t$  - or, in other words, the probability that regime  $i$  at time  $t-1$  is followed by regime  $j$  at time  $t$ . For example,  $p_{21}$  gives the probability that state 2 will be followed by state 1. Be aware that Hamilton's model assumes constant transition probabilities, which means that exogenous variables cannot affect the switching probability from one regime to another.<sup>8</sup> What is more, the Markov chain is said to be reducible if  $p_{jj} = 1$ , which means that if the process enters state  $j$ , there is no way to go to state  $i$  and so the state  $i$  is called absorbing state. On the other hand, the Markov chain is irreducible if  $p_{jj} < 1$  and  $p_{ii} < 1$ .<sup>9</sup> The unconditional probabilities that the process is in each one of the regimes according to Hamilton's derivation (1994, p. 683) are given by:

$$P(s_t = j) = \frac{1 - p_{jj}}{2 - p_{jj} - p_{ii}} \quad (9)$$

From Equation 7, the transition probabilities also provide us with the expected duration, that is the expected length the system is going to stay in a certain regime. If  $D_j$  defines the duration of regime  $j$ , then the expected duration is:

$$E(D_j) = \frac{1}{1 - p_{jj}}, \quad j = 1, \dots, k \quad (10)$$

### 3.2 Bivariate Markov Switching Model

Following the literature which offers evidence of regimes not only in the distribution of individual return series but also on pair of these (multivariate models), I consider an  $n \times 1$  vector of returns at time  $t$ ,  $r_t = (y_{1t}, y_{2t}, \dots, y_{nt})$ . Assuming that the return  $r_t$  follows a vector autoregressive (VAR) process of order zero ( $p = 0$ ) we have:

$$r_t = \mu + \varepsilon_t \quad (11)$$

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<sup>8</sup>For more details about time-varying transition probabilities models see at Diebold, Lee and Weinbach (1994).

<sup>9</sup>Here I assume that the Markov process is irreducible since, if either a single state or a block of states is absorbing, all other states will have zero steady-state probabilities.



where  $\mu$  is the  $n \times 1$  vector of intercepts and  $\varepsilon_t$  is a  $n \times 1$  of independent and identically distributed Gaussian residuals. Allowing for autoregressive components, the above equation can be defined as:

$$r_t = \mu + \sum_{i=1}^p \Phi_i r_{t-i} + \varepsilon_t \quad (12)$$

where  $\Phi_j$  is a  $n \times n$  coefficient matrix including the lags up to order  $p$ . If the times series are subject to shifts in regime, the Markov-switching vector autoregressive (MS-VAR) model might be considered. In that case, the parameters of the observed time series vector  $r_t$  depend upon the unobservable regime variable  $s_t$  and Equation 11 will become:

$$r_t = \mu_{S_t} + \varepsilon_t \quad (13)$$

where  $\mu_{S_t}$  is an  $n \times 1$  vector of state dependent intercepts,  $\mu_{S_t} = (\mu_{1S_t}, \mu_{2S_t}, \dots, \mu_{nS_t})$  and  $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \dots, \varepsilon_{nt}) \sim IID N(0, \Sigma_{S_t})$  is the vector of return innovation white noise process which has zero mean and state dependent  $n \times n$  variance covariance matrix  $\Sigma_{S_t}$ . This model denoted in the literature as the heteroskedastic and intercept regime-dependent Markov-switching MSIH( $k$ ). Additionally, we can have a constant covariance matrix over time, ( $\varepsilon_t = IID N(0, \Sigma)$ ), which refers to the homoscedastic model MSI( $k$ ). In another class of models, we can consider state dependent dynamics in the autoregressive part, or to put it differently, regime switching VAR( $p$ ) coefficients. The equivalent of Equation 12 will be the MSIAH( $k$ )-VAR( $p$ ):

$$r_t = \mu_{S_t} + \sum_{i=1}^p \Phi_{i,S_t} r_{t-i} + \varepsilon_t \quad (14)$$

where  $\Phi_{i,S_t}$  is the  $n \times n$  matrix of autoregressive coefficients associated with lag  $i \geq 1$  in state  $S_t$ .

### 3.3 Multivariate Markov Switching Model with Predictor Variables

It is natural to extend Equation 14 to allow for predictability patterns from an  $m \times 1$  vector of predictor variables  $x_{t-1}$ . If we define  $z_t = (r_t', x_t')$  as an  $(n + m) \times 1$  vector we get:

$$z_t = \begin{pmatrix} \mu_{S_t} \\ \mu_{xS_t} \end{pmatrix} + \sum_{i=1}^p \Phi_{i,S_t}^* z_{t-i} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_{xt} \end{pmatrix} \quad (15)$$

where  $\mu_{xS_t} = (\mu_{x1S_t}, \mu_{x2S_t}, \dots, \mu_{xmS_t})'$  is the intercept vector for  $x_t$  in state  $S_t$ ,  $\{\Phi_{i,S_t}^*\}_{i=1}^p$  are now  $(n+m) \times (n+m)$  matrices of autoregressive coefficients in state  $S_t$  and  $(\varepsilon'_t, \varepsilon'_{xt})' \sim N(0, \Sigma_{S_t}^*)$ , where  $\Sigma_{S_t}^*$  is an  $(n+m) \times (n+m)$  covariance matrix.

### 3.4 Estimation

If we call  $\theta (\mu_1, \mu_2, \dots, \mu_n, \sigma_1^2, \sigma_2^2, \dots, \sigma_n^2, p_{11}, p_{12}, \dots, p_{kk})$  a vector of population that includes all the estimated parameters, then the log likelihood can be estimated from:

$$L(\theta) = \sum_{t=1}^T \log f(y; \theta) \quad (16)$$

and the maximum likelihood is obtained by maximising Equation 16. This can be achieved by using the EM (Expectation-Maximisation) algorithm as proposed by Dempster, Laird and Rubin (1977) and Hamilton (1989), which is designed for a general class of models where the observed time series depends on some unobservable stochastic variables (the regimes variables  $S_t$ ). By using the EM algorithm, we are simply trying to maximise the incomplete-data log likelihood via iterative maximisation of the expected complete-data log likelihood, conditional upon the observable data.

We can describe the procedure of the algorithm in four steps. The first step, allocates an initial guess to the parameter vector  $\theta^{(0)}$ , in order to start the EM algorithm. In step two, which is the Expectation step, the algorithm produces smoothed state probabilities conditional upon  $\theta^{(0)}$ , while step three, the Maximisation step, produces an updated parameter estimate,  $\theta^{(1)}$ , conditional upon the smoothed state probabilities obtained in the previous step. If the algorithm converges (based on convergence criteria), then it will stop; otherwise we will go back to step three and repeat until convergence.<sup>10</sup>

## 4 DATA

For the purposes of this study I use monthly data of stock, bond and short-term interest rate returns for the US and UK for the period 1986:01-2010:10 - a total of 298 observations. I decided to use monthly data because of the presence of more noise at higher frequencies, such as daily data, which makes it more difficult to isolate cyclical variations and as a result obscures the analysis of

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<sup>10</sup>For more details about EM algorithm see Diebold *et al.* (1994).

the driving moments of switching behavior.<sup>11</sup> Stock and bond returns are calculated by applying the formula,  $r_t = \ln p_t - \ln p_{t-1}$ , where  $p_t$  is the asset price, while the change in short term interest rate is defined as:  $tb_t - tb_{t-1}$ , where  $tb$  denoted to the 3 month Treasury bill. All the data cited in this paper are obtained from Datastream and are expressed in domestic currencies. Tables 3 and 4 provide summary statistics for all the series under consideration.

The descriptive statistics of the data for the two countries provide very similar features. Mean stock returns in annualised terms vary from 9.72% in the case of the US to 9.96% in the case of the UK, while the mean bond returns vary from 7.08% to 8.4% respectively; volatilities - defined as the standard deviation of returns - vary between 16.21% per year for US stock returns to 16.55% for UK. On the other hand, the annualised bond return volatility varies between 4.85% (US) to 6.2% (UK). The annualised means for interest rate are between -46.32% and -27.84% and the volatilities between 113% and 72.57% for the UK and the US respectively per year. However, it is interesting to note that the mean and median changes in short term rates are non-positive, which is consistent with the fact that most of my sample period is dominated by declining short-term interest rates after the peak reached in the late 1980s and early 1990s.

What is more, all three series are characterised by negative skewness, implying that the distributions have a long left tail (left-skewed), and that the mass of the distribution is concentrated on the right. The negative skewness also suggests that large negative returns tend to occur more often than large positive ones. However, this is not true for UK bond returns, for which the skewness is very close to zero. Furthermore, the series display positive kurtosis coefficients above the Gaussian benchmark value of three for the normal distribution, suggesting that the underlying data are leptokurtotic - that is, all series have a thicker tail and a higher peak.

At this point, it is important to mention that kurtosis measures how much of the variance of the return series,  $r_t$ , is due to events that happen at the tails of the distribution (infrequent or extreme events). It is also worth noting that the kurtosis of the stock returns is larger than the kurtosis of the bond returns and interest rate returns. This difference may reflect the fact that policymakers can affect, by their actions, the bond market and the interest rates, while there are virtually no

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<sup>11</sup>In the existing literature, one can observe two distinct streams. The first (Glick and Rose (1999) and Van Rijckeghem and Weder (2001)) involves low-frequency data and has the advantage of directly incorporating fundamental variables, such as banking flows and trade. On the other hand, the majority of the empirical work (correlation, threshold, latent factor models) uses high-frequency data. Generally, one can argue that the most important difference is that the high-frequency studies tend to consider contagion as a relatively short-lived feature, whose extremes would not be captured in lower frequency applications.

such opportunities in stock markets. Surprisingly, the kurtosis for UK interest rates is higher than that of stock returns, which means that there are more events at the tails (extreme events; see part 3 or Figure 4).

The considerably large values of excess kurtosis are reflected in the high values of the JB statistics, which lead us to the rejection of the null hypothesis of a normal distribution at the 1% significance level. This means that there are significant departures from normality, which need to be taken into account when analysing financial time series and which suggest that a flexible model is required to incorporate such features.<sup>12</sup> Finally, the above-mentioned tables also report the Ljung-Box  $Q$  statistic for the fourth order serial correlation in levels and squares of returns. The  $Q$  statistic points out that, apart from the interest rate series there is no strong evidence for serial correlation in levels; however, the squared residuals do show serial correlation, suggesting strong evidence of time-varying volatility (heteroscedasticity).

Figures 1 and 3 plot the total stock and bond market index as well as the 3-month Treasury bill, while Figures 2 and 4 plot the three return series for the US and UK over the entire sample period, where the shaded areas refer to the chronologies of business cycles according to OECD for each country and represent the dates of peaks and troughs in economic activity.<sup>13</sup> All the series have been tested with regards to whether they are consistent with an  $I(1)$  process with a stochastic trend, or if they are consistent with an  $I(0)$  process, which means that it is stationary, with a deterministic trend. The results from the Augmented Dickey Fuller (ADF) test, as shown in Table 5, reject the null hypothesis of a unit root in the logarithm of price series which means that the series are all stationary.

To summarise, the features of the financial time series documented in this section seem to require nonlinear models, simply because linear models would not be able to generate data that have these features.

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<sup>12</sup>For further discussion on the reason of this departure from normality, see Pesaran (2010).

<sup>13</sup>The OECD cyclical peak and trough dates defining expansions and recessions are available at: [http://www.oecd.org/document/29/0,3746,en\\_2649\\_34349\\_35725597\\_1\\_1\\_1\\_1,00.html](http://www.oecd.org/document/29/0,3746,en_2649_34349_35725597_1_1_1_1,00.html). Turning points are reported separately for each country.

## 5 MODEL SELECTION AND SPECIFICATION TESTS

For the estimation of the appropriate MS-VAR model I need to specify: i) the number of regimes  $k$ , ii) to define the variables that I will switch - intercept (mean), autoregressive components, variance/covariance matrix - and iii) the order of the lag polynomial  $p$ . The selection of the regime-switching process is complicated, because the identification of the number of regimes cannot be effected through the usual likelihood ratio, Lagrange multiplier or Wald tests since their asymptotic distributions are non-standard.<sup>14</sup> Given the number of regimes  $k$ , a variety of model selection criteria can be applied to choose the lag length  $p$  for each model. For that reason, I perform a specification test using three information criteria, the Akaike (AIC), the Schwartz (SIC), the Hannan-Quinn (H-Q) and two likelihood ratio tests, Davies (1987) and Wolfe (1971).

A leading method for selecting one of several competing models is the method of penalised likelihood, and the model that optimises the complexity penalised likelihood is the one that fits better the data. The AIC and the SIC are applicable to general classes of models, while the H-Q is more appropriate for selecting the order of autoregressive models. More precisely, the AIC describes the trade-off between bias and variance and is based on the minimisation of the Kullback-Leibler information theory<sup>15</sup> as a measure of information lost when a particular model is used in place of the true (unknown) model. The criterion is given by:

$$AIC = 2k - 2 \ln L \tag{17}$$

where  $L$  is the maximum likelihood and  $k$  is the number of estimated parameters of the model.

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<sup>14</sup>In the literature, we can find a few different approaches to overcome this problem. For example, Davies (1987) analyses the problem of unidentified nuisance parameters and bounds the maximum of the empirical process, while Hansen (1992) extends this approach considering the likelihood function as an empirical process of the unknown parameters and bounds the asymptotic distribution of a standardised likelihood ratio statistic. On the other hand, Garcia (1998) argues that Hansen's (1992) procedure has two main drawbacks: first it is computationally heavy, and second "[it] provides a bound for the likelihood ratio statistic and not a critical value, which means that the test may be conservative". For that reason, Garcia (1999) is treating the transition probability parameters as nuisance parameters and sets the null hypothesis of the linear models to be governed by the Markov variable.

<sup>15</sup>In both probability and information theory, the Kullback-Leibler divergence (or information divergence, or relative entropy) is a natural distance measure from a 'true' probability distribution  $p$ , in this case the bias (the difference between the estimator's expectation and the true value of the parameter being estimated), to an arbitrary probability distribution  $q$ , the variance, (Kullback and Leibler (1951)). Typically  $p$  represents data, observations, or a precisely calculated probability distribution; and  $q$  represents a theory, a model, a description or an approximation of  $p$ .

The Schwartz or Bayesian information criterion (SIC or BIC) can be defined as:

$$SIC = k \ln N - 2 \ln L \tag{18}$$

where  $N$  is the number of observations in the sample. The lower the criteria, the better the specification.<sup>16</sup> Because for  $N > 8 \Rightarrow \ln N > 2$ , the SIC penalises additional parameters more heavily than the AIC. Therefore, the model order selected by the SIC is likely to be smaller than that selected by the AIC. Generally, we can say that the AIC tends to select relatively large and possibly over-parametrised models, while the SIC is in favour of small more parsimonious models.<sup>17</sup> Finally, the Hannan-Quinn criterion which can be interpreted as:

$$H - Q = 2k[\ln(\ln N)] - 2 \ln L \tag{19}$$

is in an intermediate position compared to SIC and AIC and most of the times yields to estimations which are identical to AIC.<sup>18</sup>

The most important hypothesis that someone has to test against in cases of such models is the number of different regimes  $k$  that characterise the data. Hansen (1992), Garcia (1998) and Hamilton (1996) have tried to test the linear model ( $\kappa = 1$ ) against the univariate/multivariate Markov-switching model. But this is not an easy task, according to Coe (2002), for two main reasons. First, because under the null of a single state model, AR(1) or VAR(1), some of the parameters which define the transition between the states (transition probabilities) are not identifiable.<sup>19</sup> Usually, these parameters are referred to as nuisance parameters. Second, the scores (derivatives) with respect to the nuisance parameters and the parameters associated with the second (third, fourth and so on and so forth) regime of the economy are zero under the null. This

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<sup>16</sup>Here, we have to recall that by construction information criteria illustrate an increasingly good trade-off between fit and parsimony as their values decline.

<sup>17</sup>The fact that SIC prefers very parsimonious models, containing only few parameters, has sometimes implications when we attempt to evaluate nonlinear time series models. For example, when a quite large number of parameters is needed to obtain only a slightly improved fit.

<sup>18</sup>Kapetanios (2001) found that the AIC tends to choose longer lag length in MS-AR models, whereas the SIC selects more parsimonious models. Others, like Ivanov and Kilian (2005), suggest that the Hannan-Quinn criterion is more accurate for lag length selection in a VAR models. Finally, and according to Psaradakis and Spagnolo (2006), AIC, SIC and H-Q can accurately identify the correct model structure, particularly when the sample size and parameter changes are not too small and the regime variables are correlated. They also argued that the AIC performs considerably well compared to SIC when the autoregressive order is known and we are trying to estimate the state dimension of a Markov-switching model.

<sup>19</sup>For the simple two-state model, the probabilities  $p_{12} = p_{21}$  and  $p_{22}$  of the transition matrix are not identified.

has as a result the likelihood ratio (LR) test statistic not possessing the standard Chi-squared ( $\chi^2$ ) distribution (not having the standard asymptotic distribution) and being no longer valid. This is the so-called Davies' problem in hypothesis testing. What Davies (1987) did was to derive the upper bound for the significance level of the LR test under nuisance parameters:

$$\Pr(LR > x) \leq \Pr(\chi_1^2 > x) + \sqrt{2x} \exp\left(-\frac{x}{2}\right) \left[\Gamma\left(\frac{1}{2}\right)\right]^{-1} \quad (20)$$

A modified LR test proposed by Wolfe (1971) and applied by Turner *et al.* (1989) which allow to test the hypothesis of a mixed multivariate normal distribution against the null of a simple multivariate normality. The test has the form:

$$LR = -\frac{2}{T}(T-3)(\ln L_r - \ln L_u) \underline{d}_r \chi_r^2 \quad (21)$$

where  $T$  is the number of observations,  $\ln L_r$  is the log-likelihood of the one-state restricted model and  $\ln L_u$  is the log-likelihood of the unrestricted  $k$ -state model, the Markovian switching model.

Outcomes for a range of MSIAH( $k, q$ ) models are reported in Tables 6, 7, 10, 11, 14 and 15, where the number of states  $k$  taking values of 1, 2, 3 and 4, while for the number of lags I consider only  $p = 0$  and 1.<sup>20</sup> The reason that I did not go beyond  $p = 1$ , for example  $p = 2, 3$  or 4, is that my data does not allow me to estimate large-scale models, given the fact that I have only 297 observations (when  $p = 0$ ) and 296 when I have one autoregressive component. The problem is that as the number of parameters of the Markov chain increases, the number of observations available for the estimation of the regime-dependent parameter shrink. For example, the number of estimated parameters in a univariate MSIAH(4,3) model are 32, and the saturation rate, which is the ratio between the number of observations used in estimation and the number of parameters, is 9.18. The rule of thumb, according to Guidolin and Hyde (2010), says that as the saturation ratio drops below 20 we should not have much faith in the resulting estimates, also because a large fraction of the estimates fails to be statistically significant.

Tables 6-7, 10-11 and 14-15 report the model selection results for the univariate, bivariate and

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<sup>20</sup>In the acronym MSIAH( $k, q$ ) suggested by Krolzig (1997), MS indicates Markov switching, I stands for the fact that the intercept  $\mu_{S_t}$  is regime switching, A implies the regime-dependent autoregressive (AR) component of order  $q$  and H stands for heteroscedasticity, which means that we allow the variances and covariances to vary across regimes. When  $k = 1$  and  $q = 0$  we have a single-state standard linear model, which we will use to test whether the null of a single-state can be rejected in favour of  $k > 1$ .

threevariate models for the US and UK. At a first glance, the growth in the maximised log-likelihood function and the decline in information criteria are well mentioned when moving from single-state models (for example,  $MSI(1,1)$  which is the simple Gaussian homoscedastic  $VAR(1)$ ) to two- and three-state models. Furthermore, and as mentioned earlier, the AIC tends to pick models that are over-parametrised (e.g.  $MSIAH(4,1)$ ), while the SIC is in favour of models with few parameters (e.g.  $MSIH(2,0)$ ). Finally, the H-Q is somewhere in the middle, and most of the time follows the other two criteria.

For each of the three information criteria, the above Tables boldface the three best models, following the Guidolin and Ria (2010) procedure. The most desirable model is the one that takes score from all the three information criteria. This happens in four out of six cases, for the univariate UK, the bivariate US and the threevariate US and UK model. When this is not feasible, which means that there is not a unique solution, I choose the most generic model - the one which allows for higher number of states - over those where the saturation rate is above the benchmark value of 20. To be more precise, in the univariate Markov switching framework where I am testing the presence of regime shifts in the individual stock return series, I found that a three-regime, state-dependent mean, variance, and no-autoregressive component,  $MSIH(3,0)$ , model is appropriate to describe the US and UK stock returns. On the other hand, a two state model with regime switching vector autoregressive components,  $MSIAH(2,1)$ , is the best in order to capture the possibility of regimes in the joint distribution of stock and bond returns for the US, and  $MSIH(2,0)$  for the UK. Furthermore, this former model, the  $MSIAH(2,1)$ , has been pointed out from all the information criteria as well as from the two likelihood ratio tests as the one to incorporate the addition of one predictor variable (threevariate model).

Overall, what I found is that the null hypothesis of a single state is always strongly rejected in favour of the two- or three-state models. This is clear evidence that the data seem to require the specification of Markov switching dynamics, which is consistent with the literature (Ang and Bekaert (2002), Guidolin and Timmermann (2008) and Guidolin and Hyde (2010)) who have argued that linear AR and VAR models do not appear to be able to pick up nonlinear patterns.



## 6 ESTIMATION RESULTS

### 6.1 Univariate Markov Switching Model of Stock Returns

I will begin with the interpretation of the states for the univariate case. Tables 7 and 8 provide parameter estimates (along with implied standard errors and significance levels) for the selected models fitted to monthly stock returns for the US and UK respectively. Each of the three regimes has a clear economic interpretation. The first regime is the bear state characterised by large negative mean returns and high volatility. US stock returns earn an annualised premium of -47%, while UK earn almost double (-84%) the premium. The per annum volatility varies between 21.5% for the US and 31% for the UK. The persistence of the first regime is very low, with an average duration of almost two months; while when the two stock markets leave the bear state, this is usually to switch to the bull regime, with probabilities 50% (US) and 57% (UK).

Contrary, the second regime can be described as "normal" and is characterised by positive mean returns and low volatilities. Stock returns are positive, 13.6% in the US and 17.8% in UK, and the two markets display similar risk premia, in the order of 7.2-7.7% a year. Once in the normal state, stock markets tend to stay in this state for 45 (US) and 26 (UK) months on average with probabilities reaching almost 97%, which characterises approximately 35% and 29% of the data in the long run.<sup>21</sup>

Finally, the third or bull state is associated with high mean returns (US stock returns earn an annualised premium of 31% and UK stock returns 13.4%) and above-normal volatilities (2.5%;US and 1.5%;UK). The estimates of the transition probability matrices are quite similar for the two stock markets. Starting from a bear regime, 57% of the time the UK stock market switches to a bull state (43% of the time it stays in a bear regime), while for the US stock market there is a 49% probability to switch from a bear to a bull or a 51% probability to stay in a bear regime.

To further assist with the economic interpretation of the three regimes, Figures 5 and 6 plot the smoothed state probabilities. These graphs show the most prolonged - two months in total - bear periods, such as the Black Monday of October 1987, the Persian Gulf War of 1990-91, the roaring 90's (especially during the mid-to late 90's) which lead to the dot-com bubble of March 2000, the stock market downturn of 2002, the more recent credit crunch of 2007-08, as well as the effect of the

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<sup>21</sup>Equivalently, the ergodic probabilities of the normal state are 0.35 and 0.29 respectively.

2010 sovereign debt crisis in the US and UK economies. Furthermore, and in order to see clearly how the business cycle matches the smoothed state probabilities, I have calculated the correlations between the OECD recessions and those probabilities and found that it takes values of 0.14 (bear state), -0.04 (normal state) and -0.04 (bull state) for the US and 0.08, 0.05 and -0.08 for the UK respectively for each state. This suggests that for both countries the matching is very poor. In the following sections I will examine if with the inclusion of other variables - bonds and interest rates - the estimated model will be able to better match the business cycle fluctuations.

To conclude, what I have achieved with the univariate Markov switching model is to identify the different regimes-states occurring in the US and UK stock markets from 1986 to 2010. The results clearly suggest the need of a three-state model with regimes characterised as bear, normal and bull, in order to capture the non-linearity of the stock return series.

## 6.2 Bivariate Markov Switching Model of Stock and Bond Returns

The next step is to add, alongside the stock returns, the bond returns in a bivariate Markov switching vector autoregressive framework and see if the three-state specification, that I found before, continues to fit the joint distribution of the two assets or not. Furthermore, I am particularly interested in examining how the two return series behave and interact in different phases of the economy. By allowing the asset returns to have different means, variances and correlations in different states, I simply allow for a state-dependent risk-return trade-off, with important implications for investors' asset allocation.

Tables 12 and 13 report the parameters of the selected models. Panel A presents parameter estimates for the single-state VAR(1) and VAR(0) model for the US and UK respectively. Most of the estimates are statistically significant. Bonds returns are slightly less volatile than stock returns and the simultaneous correlation between these assets varies from -0.003 for the US to 0.16 for the UK.

On the other hand, Panel B illustrates maximum likelihood estimates for the two state models. The interpretation of the regimes is relatively straightforward. The first regime is a bear state picking up periods with negative returns and very high volatility, while the second regime describes periods with positive returns and low volatility. As we move from state 1 to state 2 the risk premium on stock returns changes from -13.2% to 21.2% for the US and from -11.5% to 20% for the UK

per annum, while the volatility declines from 2.7% to 0.9% in the case of the US and from 3.1% to 0.9% for the UK. At the same time, US bond returns in regime 1 earn a risk premia of 8.2% and UK bonds of 7% on annualised basis with volatilities almost two times higher than those in state 2.<sup>22</sup>

The estimated transition matrix shows that the markets will remain in the bear state if they are in this, with a probability of the range of 87-91%, while the average duration varies between 8 to 12 months for the UK and US respectively, indicating that the bear state is moderately persistent. This regime, as we can see in Figures 7 and 8, appears around the stock market crash of October 1987, the Kuwait invasion of August 1990, at the end of the 90's as a consequence of the Asian flu, at the beginning of the 00's with the tech bubble, the global economic recession of 2007-2008, but also during 2010 and until the end of my sample period which captures the impact that the debt crisis has in the two economies. At this point it is worthwhile to mention that there is large improvement in the correlation between the business cycle dates and the smoothed probabilities for the bivariate US model (0.28).<sup>23</sup> Opposite to the bear state, the bull state is more consistent (average duration 16 and 25 months for the US and UK respectively) and as a result, around 70% of any long sample ought to be generated by this state.

The estimated correlations between the two asset returns in the US and UK exhibit similar patterns over the two regimes. In the bear state the correlation varies from a negative -0.26 for the US and -0.45 for the UK to a positive 0.36 and 0.57 respectively in the bull regime.<sup>24</sup> The intuition behind this relationship is the so-called "flight-to-quality" phenomenon. In turbulent financial market periods investors tend to become more risk averse, thereby prompting shifts of funds out of the stock market into safer asset classes, such as long-term government bonds. This increase in the equity risk premium, on the one side, and decrease in the bond risk premium on the other, forces stock and bond prices to move in the opposite direction during periods of market turmoil.

There are two important findings which arise from the bivariate stock-bond Markov switching

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<sup>22</sup>These changes in the first two moments (mean-variance) of the distribution of asset returns are the base of tests of contagion as shown by Dungey *et al.* (2005).

<sup>23</sup>The correlation that Guidolin and Timmermann (2006) found was at the level of 0.32.

<sup>24</sup>Guidolin and Timmermann (2005) using an MSIH(3,0) model, found that the correlation ranges from -0.45 in the bear state to 0.55 in the bull. Similarly, Guidolin and Timmermann (2006), after estimating a MSIAH(4,0), they concluded that the correlation between large cap and bond returns varies from 0.37 in the recovery state to -0.40 in the crash.

analysis that I have performed. The first one has to do with the number of regimes. In the previous section I found that a more complicated three-state model is appropriate to describe the univariate stock return series, while a simple two-regime model is needed to capture the dynamics of the joint distribution. This implies that bond returns appear to be governed by a different process than stocks. The second and most important result has to do with the correlation between the two assets in times of falling stock markets. From an investor's point of view, knowing that the current state of the economy is a persistent bull market will make the investor more attracted to risky assets (stock) than if he/she was in a bear state. Likewise, when the stock market volatility is higher in turmoil times than in tranquil times, investing in equity assets is less attractive than investing in the bond market. This switch from stocks to bonds that takes place during times of sluggish economy, is referred to as flight-to-quality.

### **6.3 Threevariate Markov Switching Model With One Predictor Variable**

In this section I pose three questions. First, whether the linkages between monetary policy and financial returns are stable or not over time; second how the inclusion of one predictor variable, namely interest rates, will affect the behaviour and consequently the correlation between stock and bond returns in the bear state; and third, if there are predictability patterns of either stocks or bonds based on interest rates.

After estimating many different MS models, I have concluded that the best model specification for the two countries under examination is the MSIAH(2,1). Estimation results are reported in Tables 16 and 17, where the first panel (Panel A) presents the estimated parameters of the linear model, and the second panel (Panel B) illustrates the two-state specification with one autoregressive component. For both countries, in the linear model the intercepts are positive and statistically significant, except for the interest rate.

Looking at Panel A, the implications for the predictability of asset returns are rather interesting: US stock returns are weakly predictable using bonds (coefficient -0.0579), and strongly predictable using interest rate which forecasts positive returns (coefficient 0.5442). UK stock returns are also positively high and statistically significant predicted from past interest rates (coefficient 1.0717). Furthermore, interest rates, which are highly persistent (coefficient 0.4210 in US and 0.3944 in the UK) predict statistically significant negative returns on bonds (-4.5304 and -5.6707 for the US and

UK respectively). This result is also consistent with the negative correlation between bonds and interest rates, since as it is expected bonds move inversely to interest rates. Finally, the VAR model suggest low and negative stock-bond correlation for the US (-0.0028), and positive (0.16) for the UK.

Switching to the two-regime model (panel B), regimes can be interpreted as bear (state 1) and bull (state 2) states. For the US, regime 1 continues to pick up market crashes, characterised by negative, double-digit mean returns for stocks (-23.4%) and interest rates (-75.2%) on an annualised basis, and highly positive bond returns (10.3%). The probability of regime 1 taking values is very close to one around many well-known episodes - as in the bivariate case - with low returns and high volatility, but now the bear state is slightly less persistent with its average duration exceeding 9 months. Furthermore, the correlation between interest rate and stock returns is positive and statistically significant at 5% significance level, indicating that in periods of falling stock markets, interest rates respond negatively. The negative correlation between stock-bond returns (-0.1953) in this state implies the existence of flight-to-quality effects from stocks to bonds.

On the other hand, regime 2 is a bull state in which the annualised mean returns are positive and the volatilities are low. This state is highly persistent, lasting on average almost 24 months, while the ergodic probability confirms that roughly 72% of the sample period is captured by this regime. The stock market rally appears to have a positive and statistically significant effect on the correlation between stocks and bonds. This is in line with the expectations, since in bull markets both equities and bonds are likely to move up together, thus raising their correlation. In this state we also observe a negative correlation between stock returns and interest rates, indicating that lower rates typically mean higher stock returns and vice versa.<sup>25</sup>

Turning now to the UK, the interpretation of the two states is slightly different. The regime one is a bear state in which the annualised expected returns are positive (2.76% for stock, 5.4% for bond, and 1.7% for interest rate) and the volatilities are low (16.2%, 5% and 34.1% for the three assets respectively). This regime is highly persistent with an average duration of 20 months with the probability of remaining in this state reaching 95%. Consequently, the bear state characterises

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<sup>25</sup>There are two possible explanations of this negative relationship. The first one links to the macroeconomic conditions, since low interest rates make the cost of borrowing money cheaper and increase investment. The second is the asset attractiveness. As interest rates are high, bank deposit rates rise and new issues of government securities are made at a higher premium rate. As the relative reward for investing in stocks falls, investors move money out of the stock market into government bonds.

the 71% of any long sample. Contrary, the second regime is a bull state with positive mean returns and higher than the bear state volatilities on all assets. The probability that the economy will stay in this regime is 88% and the duration is 8 months. The estimated correlation matrix in Table 17 implies that the stock-bond correlation is negative (-0.11) in the bear state indicating that investors prefer to switch from stock to bonds in sluggish economy times.

For both countries, estimates of the autoregressive matrices suggest that the effect of changes in the interest rate on asset returns continues to be strong in the two-state model. Therefore, the inclusion of the short-term interest rate does not weaken the evidence of multiple states. In both regimes, the autoregressive coefficients indicate substantial predictability of bonds returns, and to a lesser extend of stock returns. To be more precise, interest rates forecast negative and statistically significant bond returns in both states, since the interest rate affects the price of bonds through changing the discount rate, causing bond prices to be inversely related to interest rate changes.

Regarding the matching of the bear state smoothed probabilities with the OECD business cycle dates, the correlation for the US increased even more than before, to the level of 0.31. This means that interest rates add additional explanatory power to the model and further improve further the specification. But this does not happened in the UK, where the correlation dropped to zero. A possible explanation of that could be the statistical behaviour that UK interest rates show. Surprisingly, the kurtosis coefficient (7.9406) is very large, six times higher than the corresponding value for the US (1.2520). In other words, there are extreme and infrequent events that happen on the tails of the distribution. This is true if we look at interest rate graphs in Figures 3 and 4, where we can see how volatile is the series is. Also, the large value of excess kurtosis is reflected in the huge value of the Jarque-Bera statistic (814.2404) reported in Table 3 which strongly rejects the null hypothesis of a normal distribution. Finally, another interesting thing is the divergence of the minimum (-1.82) and maximum (1.28) compared to what normally we would have expected, which is the minimum and maximum to fall in the region of  $\pm 2.33 \times \text{St.Dev} = \pm 0.76$ .<sup>26</sup>

## 7 CONCLUSION

This paper examines the comovement and correlation between stock and bond returns, as well as the impact that macroeconomic variables, and especially short-term interest rates, have on this

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<sup>26</sup>With 99% confidence interval.

relationship. As a starting point, I have tried to capture non-linearities in the joint process of stock and bond returns. By using Markov-Switching models, I found evidence that two regimes are required to explain the time-variation in the mean, variance and correlation between the two asset classes. The results suggest that the two-state specification with a high-volatility regime with negative mean returns and a persistent bull state with positive mean returns and low levels of volatility, is able to capture important features of the US and UK stock and bond returns.

Moreover, my empirical findings indicate that in the bear state the correlation between the two assets is negative and switches to a positive as the economy moves to the bull state. This relationship can be explained taking into consideration the fact that when the stock market is falling investors tend to become more risk averse, thereby prompting shifts of funds out of the stock market into safer asset classes, such as long-term government bonds. This increase/decrease in the equity/bond risk premium forces stock and bond returns to move in the opposite direction during periods of market turmoil and give rise to "flight-to-quality" phenomena. This result has very important implications on many levels. First, it is likely to provide useful and valuable information for investors behaviour in normal times and under extreme market conditions; and second, this behaviour can contribute to the stability or instability of the financial system which is why it is important for regulators and policy makers.

Furthermore, my analysis demonstrates that the dynamic linkages between financial markets and macroeconomy have been unstable over time, a finding which supports the idea of predictability patterns from interest rates. In this sense, stock and bond returns become highly predictable using past-lagged values of short-term interest rates. This is also evidence that Markov switching models are able to capture the time-varying and unstable nature of the links between monetary policy and equity markets, and thus provide a useful support to optimal decisions.

There is a long list of several extensions that would be accommodated in the framework and which are likely to improve performance. First, while I allowed only the first (mean) and second (variance) moments of returns to switch between the states, another possibility would be to allow for higher moments, such as skewness and kurtosis. The intuition behind this is that higher-order moments add considerable explanatory power compared to standard mean-variance cases, as suggested by Harvey and Siddique (2000) and Guidolin and Timmermann (2008). In fact, both higher-order preferences and regimes turn out to play an important role in international asset

allocation since they can affect investors' decisions. Also, the descriptive statistics in Tables 3 and 4 highlight the possibility of additional contagious channels operating through higher order co-moments.

An interesting issue that goes beyond the analysis of the current paper is the out-of-sample exercise. What matters for a model is not its ability to produce an accurate in-sample fit, but rather its out-of-sample performance. Forecasting stock returns is a fascinating endeavor with a long history. While there is sufficient in-sample evidence that stock returns are predictable using a variety of economic variables (dividend-price ratio, earning-price ratio, nominal interest rate and inflation rate among others), there is also evidence - Goyal and Welch (2003 and 2008) - that the predictive ability of these variables does not hold up in out-of-sample forecasting exercises. In addition, understanding the nature of stock return forecastability in the data helps to produce more realistic asset pricing models, but also has important implications for tests of market efficiency. Recent studies, such as Guidolin and Timmermann (2005 and 2009), have found that regime-switching models may prove extremely useful to forecast especially over low frequencies, such as monthly data.



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# APPENDIX A

**Table 1**  
**Data**

Variable	Source	Mnemonic/Code
Stock Return $\ln(p_t) - \ln(p_{t-1})$	Total Stock Market Index, Datastream	TOTMKUK(RI), TOTMKUS(RI)
Bond Return $\ln(p_t) - \ln(p_{t-1})$	Total Bond Return Index Datastream	UKMGUKRI, USMGUSRI
Change in Sort-term interest rate $tb_t - tb_{t-1}$	3 Month Treasury Bill ( $tb$ ), Datastream	UKI60C, USI60C

Notes: The total market index covers all the sectors in each country. RI stands for return index and presents the theoretical growth in value of a notional share holding, the price of which is that of the selected price index. This holding is deemed to return a daily dividend, which is used (re-invested) to purchase new (additional) units of the stock at the current price. The gross dividend is used:  $RI_t = RI_{t-1} \times \frac{PI_t}{PI_{t-1}} \times \left(1 + \frac{DY \times f}{n}\right)$ , where  $RI_t$  and  $RI_{t-1}$  are the return indexes on day  $t$  and  $t-1$  respectively,  $PI_t$  and  $PI_{t-1}$  are the price indexes on day  $t$  and  $t-1$  respectively,  $DY$  is the dividend yield of the price index,  $f$  is the grossing factor (normally 1) - is the dividend yield is a net figure rather than gross,  $f$  is used to gross up the yield, and  $n$  is the number of days in financial year (normally 260)  $\times 100$ .

**Table 2**  
**Business Cycle Peak and Trough Dates**

United States	United Kingdom
P-T	P-T
1984:07-1986:09	-
1988:11-1991:04	1988:11-1992:05
1994:12-1996:02	1994:10-1999:01
2000:05-2001:12	2000:11-2003:04
2002:09-2005:08	2004:04-2005:09
2008:02-2009:05	2008:02-2009:05

Notes: The chronologies of turning points are obtained from OECD and the main reference series used are industrial production (IIP) - including all industry sectors except construction- and the Gross Domestic Product (GDP) to supplement the IIP series. A recession is a period between a peak and a trough. An expansion is a period between a trough and a peak.

**Table 3**  
**Summary Statistics for United States**

Variables	Stock	Bond	Interest Rate
Maximum	0.1256	0.0547	0.4500
Minimum	-0.2325	-0.0478	-0.8300
Mean	0.0081	0.0059	-0.0232
Median	0.0142	0.0065	-0.0100
St. Dev.	0.0468	0.0140	0.2095
Skewness	-1.0862	-0.1792	-0.8461
Kurtosis	6.0477	3.8393	4.2520
JB statistic	173.3568***	10.3092***	54.8371***
LB(4)	3.6349	8.2980*	137.88***
LB(4) squares	114.96***	101.53***	100.22***
Observations	297	297	297

Notes: \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

**Table 4**  
**Summary Statistics for United Kingdom**

Variables	Stock	Bond	Interest Rate
Maximum	0.1400	0.0710	1.2800
Minimum	-0.2963	-0.0678	-1.8200
Mean	0.0083	0.0070	-0.0386
Median	0.0129	0.0075	-0.0100
St. Dev.	0.0478	0.0179	0.3260
Skewness	-1.2295	-0.0032	-0.8281
Kurtosis	8.2851	4.3215	10.9406
JB statistic	420.5041***	21.6137***	814.2404***
LB(4)	8.7843*	5.9025	86.410***
LB(4) squares	114.33***	107.81***	103.54***
Observations	297	297	297

Notes: \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

**Table 5**  
**Augmented Dickey-Fuller (ADF) Unit Root Test for the Three Series for US and UK**

No of lags	US			UK		
	Stock	Bond	Interest rate	Stock	Bond	Interest rate
0	-15.6790***	-16.0630***	-9.9500***	-15.7043***	-15.2319***	-10.1467***
1	-12.1957***	-13.8850***	-8.3855***	-12.8134***	-12.8194***	-9.2403***
2	-9.37318***	-10.2636***	-6.1068***	-10.2925***	-10.2271***	-8.5683***
3	-8.3159***	-8.4704***	-5.8986***	-7.7915***	-8.8329***	-6.5473***
4	-7.1315***	-8.3214***	-5.3338***	-7.4589***	-7.9945***	-6.4386***

Notes: \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

**Table 6**  
**Model Selection Results for Univariate US Stock Returns**

Model	Number of parameters	Log likelihood		Davies linearity test		Wolfe linearity test		AIC	BIC	H-Q	Saturation rate
		Log likelihood	test	test	test						
<i>Base model: MSI(1,0)</i>											
MSI(2,0)	5	487.9242	41.1824***	40.8550***	-3.2789	-3.2665	-3.2739	-3.2665	-3.2739	148.50	
MSIH(2,0)	6	508.5601	518.9713	62.2049***	-3.3903	-3.3281	-3.3654	-3.3281	-3.3654	59.40	
MSI(3,0)	10	516.4712	57.2045***	56.5174***	-3.4544	<b>-3.3797</b>	<b>-3.4245</b>	<b>-3.3797</b>	<b>-3.4245</b>	49.50	
<b>MSIH(3,0)</b>	12	529.4752	83.2125***	82.2627***	-3.4106	-3.2862	-3.3608	-3.2862	-3.3608	29.70	
MSI(4,0)	17	516.3069	56.8759***	56.1921***	-3.4847	<b>-3.3354</b>	<b>-3.4249</b>	<b>-3.3354</b>	<b>-3.4249</b>	24.75	
MSIH(4,0)	20	541.7562	107.7747***	106.5766***	-3.3623	-3.1509	-3.2777	-3.1509	-3.2777	17.47	
<i>Base model: MSI(1,1)</i>											
MSIA(2,1)	7	487.9375	509.2939	42.8199***	-3.2833	-3.2584	-3.2733	-3.2584	-3.2733	98.66	
MSIAH(2,1)	8	519.2973	62.8267***	62.0861***	-3.3939	-3.3066	-3.3589	-3.3066	-3.3589	42.28	
MSIA(3,1)	13	514.9255	53.9760***	53.4308***	-3.4547	<b>-3.3550</b>	-3.4148	<b>-3.3550</b>	-3.4148	37.00	
MSIAH(3,1)	15	533.7625	91.7572***	90.7243***	-3.3711	-3.1716	-3.3265	-3.1716	-3.3265	22.76	
MSIA(4,1)	21	524.5477	73.2204***	72.4809***	-3.5052	-3.3181	<b>-3.4303</b>	-3.3181	<b>-3.4303</b>	19.73	
MSIAH(4,1)	24	541.3958	107.0237***	105.8367***	-3.3753	-3.0636	-3.2505	-3.0636	-3.2505	14.09	

Notes: \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

**Table 7**  
**Model Selection Results for Univariate UK Stock Returns**

Model	Number of parameters	Log likelihood		Davies linearity test		Wolfe linearity test		AIC	BIC	H-Q	Saturation rate
		Log likelihood	test	test	test						
<i>Base model: MSI(1,0)</i>											
MSI(2,0)	5	481.7581	498.7641	34.0118***	-3.2374	-3.2249	-3.2324	-3.2249	-3.2324	148.50	
MSIH(2,0)	6	509.4175	55.3187***	54.7601***	-3.3250	-3.2628	-3.3001	-3.2628	-3.3001	59.40	
MSI(3,0)	10	499.0966	34.6768***	34.3268***	-3.3900	<b>-3.3154</b>	<b>-3.3601</b>	<b>-3.3154</b>	<b>-3.3601</b>	49.50	
<b>MSIH(3,0)</b>	12	520.3728	77.2293***	76.4494***	-3.2936	-3.1692	-3.2438	-3.1692	-3.2438	29.70	
MSI(4,0)	17	499.4883	35.4603***	35.1022***	-3.4234	<b>-3.2741</b>	<b>-3.3636</b>	<b>-3.2741</b>	<b>-3.3636</b>	24.75	
MSIH(4,0)	20	524.2885	85.0608***	84.2017***	-3.2491	-3.0377	-3.1644	-3.0377	-3.1644	17.47	
<i>Base model: MSI(1,1)</i>											
MSIA(2,1)	7	480.9095	487.5668	11.6463**	-3.2558	-3.1471	-3.2963	-3.1471	-3.2963	14.85	
MSIAH(2,1)	8	509.6420	55.7967***	56.8846***	-3.2471	-3.1598	-3.2121	-3.1598	-3.2121	42.28	
MSIA(3,1)	13	510.7602	59.7014***	59.0984***	-3.3895	<b>-3.2897</b>	-3.2631	<b>-3.2897</b>	-3.2631	37.00	
MSIAH(3,1)	15	517.3327	71.1781***	72.1107***	-3.3430	-3.1435	-3.3193	-3.1435	-3.3193	22.76	
MSIA(4,1)	21	515.7931	69.7672***	69.0626***	<b>-3.3941</b>	-3.2071	-3.3193	-3.2071	-3.3193	19.73	
MSIAH(4,1)	24	517.6253	73.4316***	72.6899***	-3.3162	-3.0045	-3.1914	-3.0045	-3.1914	14.09	

Notes: \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.



**Table 8**  
**Estimation Results of the Univariate Regime Switching Model for US Stock Returns**

Stock returns			
Model	MSIH(3,0)		
<i>1. Intercept</i>			
$\mu_1$	-0.0391 (0.0215)		
$\mu_2$	0.0114*** (0.0023)		
$\mu_3$	0.0259*** (0.0072)		
<i>2. Volatilities</i>			
State 1	0.0621*** (0.0089)		
State 2	0.0222*** (0.0017)		
State 3	0.0389*** (0.0044)		
<i>3. Transition probabilities</i>	State 1	State 2	State 3
State 1	0.5093 (17.6115)	$8.650 \times 10^{-5}$ (12.3155)	0.4906
State 2	0.0217 (0.8224)	0.9776 (0.9475)	0.0006
State 3	0.1927 (0.8125)	0.0175 (0.7454)	0.7898
<i>4. State Duration</i>			
State 1	2.04 {0.1932}		
State 2	44.65 {0.3548}		
State 3	4.76 {0.4520}		

Notes: Standard errors in parentheses. \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

**Table 9**  
**Estimation Results of the Univariate Regime Switching Model for UK Stock Returns**

Stock returns			
Model	MSIH(3,0)		
<i>1. Intercept</i>			
$\mu_1$	-0.0702 (0.0509)		
$\mu_2$	0.0149*** (0.0024)		
$\mu_3$	0.0112*** (0.0043)		
<i>2. Volatilities</i>			
State 1	0.0897** (0.0247)		
State 2	0.0207*** (0.0017)		
State 3	0.0468*** (0.0032)		
<i>3. Transition probabilities</i>	State 1	State 2	State 3
State 1	0.4308 (21.5962)	0.0002 (0.6762)	0.5689
State 2	$7.310 \times 10^{-5}$ (1.2136)	0.9611 (0.9810)	0.0388
State 3	0.0399 (19.8702)	0.0168 (0.7199)	0.9432
<i>4. State Duration</i>			
State 1	1.76 {0.0467}		
State 2	25.70 {0.2881}		
State 3	17.62 {0.6652}		

Notes: Standard errors in parentheses. \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

**Table 10**  
**Model Selection Results for Multivariate US Stock and Bond Returns**

Model	Number of parameters	Log likelihood	Davies linearity test	Wolfe linearity test	AIC	BIC	H-Q	Saturation rate
<i>Base model: MSI(1,0)</i>								
	5	1333.2020			-8.9647	-8.9398	8.9196	118.80
MSI(2,0)	9	1344.2774	22.1403***	21.8083***	-8.9918	-8.8798	-8.9470	66.00
MSIH(2,0)	12	1380.7239	95.0332***	93.9651***	-9.2170	<b>-9.0678</b>	<b>-9.1572</b>	49.50
MSI(3,0)	15	1366.9739	67.5332***	66.7428***	-9.1042	-8.9176	-9.0295	39.60
MSIH(3,0)	21	1391.4444	116.4743***	115.1895***	-9.2286	<b>-8.9674</b>	-9.1240	28.28
MSI(4,0)	23	1381.6998	96.9851***	95.8972***	-9.1495	-8.8634	-9.0350	25.82
MSIH(4,0)	32	1406.9425	147.4705***	145.8727***	-9.2589	-8.8609	-9.0995	18.56
<i>Base model: MSI(1,1)</i>								
	9	1340.1360			-9.0144	-8.9396	-8.9492	65.77
MSIA(2,1)	17	1373.3093	66.4517***	65.6765***	-9.1643	-8.9523	-9.0794	34.82
<b>MSIAH(2,1)</b>	20	1393.7115	107.2562***	106.0688***	<b>-9.2818</b>	<b>-9.0325</b>	<b>-9.1820</b>	29.60
MSIA(3,1)	27	1387.0696	93.9724***	92.9191***	-9.1897	-8.8530	-9.0549	21.92
MSIAH(3,1)	33	1412.4152	144.6634***	143.0984***	<b>-9.3204</b>	-8.9089	<b>-9.1556</b>	17.93
MSIA(4,1)	39	1395.9869	111.7018***	110.5736***	-9.1622	-8.6635	-8.9624	15.17
MSIAH(4,1)	48	1428.0213	175.8757***	173.9953***	<b>-9.3245</b>	-8.7260	-9.0849	12.33

Notes: \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

**Table 11**  
**Model Selection Results for Multivariate UK Stock and Bond Returns**

Model	Number of parameters	Log likelihood	Davies linearity test	Wolfe linearity test	AIC	BIC	H-Q	Saturation rate
<i>Base model: MSI(1,0)</i>								
	5	1259.8180			-8.4701	-8.4452	-8.4251	118.80
MSI(2,0)	9	1268.1639	16.6913***	16.5232***	-8.4792	-8.3673	-8.4344	66.00
<b>MSIH(2,0)</b>	12	1305.3888	91.1412***	90.2211***	-8.7097	<b>-8.5604</b>	<b>-8.6499</b>	49.50
MSI(3,0)	15	1294.0772	68.5180***	67.8264***	-8.6133	-8.4268	-8.5386	39.60
MSIH(3,0)	21	1314.8961	110.1558***	109.0436***	-8.7131	<b>-8.4519</b>	-8.6085	28.28
MSI(4,0)	23	1299.5182	79.3999***	78.5985***	-8.5961	-8.3100	-8.4816	25.82
MSIH(4,0)	32	1334.2402	148.8440***	147.3411***	<b>-8.7693</b>	-8.3713	<b>-8.6100</b>	18.56
<i>Base model: MSI(1,1)</i>								
	9	1262.1270			-8.4873	-8.4125	-8.4221	65.77
MSIA(2,1)	17	1274.3566	24.4584***	24.2122***	-8.4957	-8.2837	-8.4108	34.82
MSIAH(2,1)	20	1306.6110	88.9672***	88.0694***	-8.6933	<b>-8.4440</b>	-8.5935	29.60
MSIA(3,1)	27	1305.8444	87.4342***	86.5517***	-8.6408	-8.3042	-8.5061	21.92
MSIAH(3,1)	33	1337.6668	151.0789***	149.5537***	<b>-8.8153</b>	-8.4039	<b>-8.6506</b>	17.93
MSIA(4,1)	39	1327.9302	131.6064***	130.2772***	-8.6887	-8.1651	-8.4791	15.17
MSIAH(4,1)	48	1343.9027	163.5507***	161.8995***	<b>-8.7561</b>	-8.1577	-8.5165	12.33

Notes: \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

**Table 12**  
**Estimation Results of the Multivariate Regime Switching Model for US Stock and Bond**

		<b>Returns</b>	
		Stock	Bond
<b>A.</b>	<b>Model</b>	<b>VAR(1)</b>	
<i>1. Mean return</i>			
	<i>c</i>	0.0065** (0.0029)	0.0058*** (0.0008)
<i>2. VAR(1) matrix</i>			
	Stock	0.0917 (0.0580)	-0.0575*** (0.0169)
	Bond	0.1142 (0.1931)	0.0753 (0.0565)
<i>3. Correlations/Volatilities</i>			
	Stock	0.0021***	
	Bond	-0.0033**	0.0001***
<b>B.</b>	<b>Model</b>	<b>MSIAH(2,1)</b>	
<i>4. Intercept</i>			
	$\mu_1$	-0.0111 (0.0079)	0.0068*** (0.0020)
	$\mu_2$	0.0177*** (0.0027)	0.0051*** (0.0009)
<i>5. VAR(1) matrix</i>			
	State 1		
	Stock	0.2429** (0.1235)	-0.0834*** (0.0321)
	Bond	0.5119 (0.4395)	-0.1442 (0.1187)
	State 2		
	Stock	-0.2425*** (0.0838)	-0.0718*** (0.0266)
	Bond	0.3041 (0.2240)	0.2749*** (0.0757)
<i>6. Correlations/Volatilities</i>			
	State 1		
	Stock	0.0628*** (0.0050)	
	Bond	-0.2634*** (0.0019)	0.0170*** (0.0008)
	State 2		
	Stock	0.0321*** (0.0013)	
	Bond	0.3631*** (0.0019)	0.0112*** (0.0005)
<i>7. Transition probabilities</i>			
		State 1	State 2
	State 1	0.9134 (0.5116)	0.0866
	State 2	0.0405 (0.4465)	0.9595
<i>8. State Duration</i>			
	State 1	11.55 {0.3186}	
	State 2	24.70 {0.6814}	

Notes: The VAR coefficient estimates should be read in the following way: the coefficient illustrates the impact of a change in the variable listed in the corresponding column on the variable listed in the corresponding row. Standard errors are reported in parentheses. \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

**Table 13**  
**Estimation Results of the Multivariate Regime Switching Model for UK Stock and Bond**

		<b>Returns</b>	
		Stock	Bond
<b>A.</b>	<b>Model</b>	<b>VAR</b>	
<i>1. Intercept</i>			
	$c$	0.0083*** (0.0027)	0.0070*** (0.0010)
<i>2. Correlations/Volatilities</i>			
	Stock	0.0022***	
	Bond	0.1677**	0.0003***
<b>B.</b>	<b>Model</b>	<b>MSIH(2,0)</b>	
<i>3. Intercept</i>			
	$\mu_1$	-0.0096 (0.0091)	0.0058*** (0.0021)
	$\mu_2$	0.0167*** (0.0028)	0.0076* (0.0015)
<i>4. Correlations/Volatilities</i>			
	State 1		
	Stock	0.0623*** (0.0061)	
	Bond	-0.4511*** (0.0016)	0.0150*** (0.0018)
	State 2		
	Stock	0.0362*** (0.0013)	
	Bond	0.5696*** (0.0020)	0.0190*** (0.0010)
<i>5. Transition probabilities</i>			
		State 1	State 2
	State 1	0.8718 (0.6170)	0.1282
	State 2	0.0622 (0.5012)	0.9378
<i>6. State duration</i>			
	State 1	7.80 {0.3266}	
	State 2	16.08{0.6734}	

Notes: Standard errors are reported in parentheses. \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

**Table 14**  
**Model Selection Results for US Stock, Bond and Interest Rate Returns**

Model	Number of parameters	Log likelihood	Davies linearity test	Wolfe linearity test	AIC	BIC	H-Q	Saturation rate
<i>Base model: MSI(1,0)</i>	9	1384.5880			-9.3036	-9.2663	-9.2184	99.00
MSI(2,0)	14	1432.5465	96.0251***	94.9482***	-9.5525	-9.3784	-9.4828	63.64
MSIH(2,0)	20	1476.1913	183.3147***	181.3562***	-9.8060	<b>-9.5573</b>	-9.7064	44.55
MSI(3,0)	21	1452.0347	135.0017***	133.5310***	-9.6366	-9.3754	-9.5320	42.42
MSIH(3,0)	33	1511.0278	252.9878***	250.3255***	-9.9530	-9.5426	-9.7887	27.00
MSI(4,0)	30	1480.0559	191.0441***	189.0073***	-9.7647	-9.3916	-9.6153	29.70
MSIH(4,0)	48	1542.9954	316.9231***	313.6150***	-10.0673	-9.4703	<b>-9.8283</b>	18.56
<i>Base model: MSI(1,1)</i>	18	1454.8440			-9.7489	-9.5993	-9.6186	49.33
MSIA(2,1)	32	1507.4810	105.3767***	104.2107***	-9.9695	<b>-9.5705</b>	-9.8097	27.75
<b>MSIAH(2,1)</b>	38	1529.8233	150.0612***	148.4440***	<b>-10.0799</b>	<b>-9.6061</b>	<b>-9.8902</b>	23.36
MSIA(3,1)	48	1533.5958	157.6062***	155.9128***	-10.0378	-9.4394	-9.7982	18.50
MSIAH(3,1)	60	1555.5803	201.5752***	199.4377***	<b>-10.1053</b>	-9.3572	-9.8058	14.80
MSIA(4,1)	66	1546.7859	183.9865***	182.0266***	-10.0053	-9.1825	-9.6759	13.45
MSIAH(4,1)	84	1604.2749	298.9643***	295.8433***	<b>-10.2721</b>	-9.2249	<b>-9.8528</b>	10.57

Notes: \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

**Table 15**  
**Model Selection Results for UK Stock, Bond and Interest Rate Returns**

Model	Number of parameters	Log likelihood	Davies linearity test	Wolfe linearity test	AIC	BIC	H-Q	Saturation rate
<i>Base model: MSI(1,0)</i>	9	1187.3060			-7.9751	-7.9378	-7.8899	99.00
MSI(2,0)	14	1230.8223	87.0331***	86.1536***	-8.1941	-8.0200	-8.1244	63.64
MSIH(2,0)	20	1351.5294	328.4473***	325.1295***	-8.9665	<b>-8.7178</b>	-8.8670	44.55
MSI(3,0)	21	1253.3123	132.0131***	130.6793***	-8.2984	-8.0372	-8.1938	42.42
MSIH(3,0)	33	1376.0304	377.4493***	373.6366***	-9.0440	<b>-8.6336</b>	-8.8797	27.00
MSI(4,0)	30	1257.8849	141.1583***	139.7321***	-8.2686	-7.8955	-8.1192	29.70
MSIH(4,0)	48	1394.4720	414.3324***	410.1472***	-9.0672	-8.4702	-8.8282	18.56
<i>Base model: MSI(1,1)</i>	18	1251.7750			-8.3768	-8.2272	-8.2465	49.33
MSIA(2,1)	32	1305.7293	107.9094***	106.8187***	-8.6063	-8.2073	-8.4465	27.75
<b>MSIAH(2,1)</b>	38	1397.4069	291.2646***	288.3220***	<b>-9.1852</b>	<b>-8.7114</b>	<b>-8.9955</b>	23.36
MSIA(3,1)	48	1362.2606	220.9719***	218.7394***	-8.8801	-8.2817	-8.6405	18.50
MSIAH(3,1)	60	1446.3116	389.0740***	385.1436***	<b>-9.3670</b>	-8.6189	<b>-9.0675</b>	14.80
MSIA(4,1)	66	1394.5955	285.6418***	282.7560***	-8.9770	-8.1541	-8.6475	13.45
MSIAH(4,1)	84	1469.7178	435.8863***	431.4832***	<b>-9.3630</b>	-8.3157	<b>-8.9437</b>	10.57

Notes: \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

**Table 16**  
**Estimation Results of the Multivariate Regime Switching Model for US Stock, Bond and**

		<b>Interest Rate Returns</b>		
		Stock	Bond	Interest rate
<b>A.</b>	<b>Model</b>	<b>VAR(1)</b>		
<i>1. Mean return</i>				
	<i>c</i>	0.0065** (0.0029)	0.0058*** (0.0008)	0.0091 (0.0109)
<i>2. VAR(1) matrix</i>				
	Stock	0.0931 (0.0581)	-0.0579*** (0.0170)	0.5442*** (0.2117)
	Bond	0.0948 (0.1985)	0.0797 (0.0580)	-4.5304*** (0.7224)
	Interest rate	-0.0058 (0.0133)	0.0013 (0.0039)	0.4210*** (0.0485)
<i>3. Correlations/Volatilities</i>				
	Stock	0.6400***		
	Bond	-0.0028**	0.0547**	
	Interest rate	0.0822**	-0.2241***	8.4746***
<b>B.</b>	<b>Model</b>	<b>MSIAH(2,1)</b>		
<i>4. Intercept</i>				
	$\mu_1$	-0.0195* (0.0101)	0.0086*** (0.0024)	-0.0627** (0.0300)
	$\mu_2$	0.0194*** (0.0026)	0.0049*** (0.0009)	0.0434*** (0.0123)
<i>5. VAR(1) matrices</i>				
	State 1			
	Stock	0.2166* (0.1165)	-0.0772** (0.0326)	0.6929* (0.3806)
	Bond	0.4165 (0.4140)	-0.0847 (0.1235)	-4.5490*** (1.4867)
	Interest rate	-0.0409 (0.0303)	0.0081 (0.0078)	0.3269*** (0.0972)
	State 2			
	Stock	-0.2503*** (0.0755)	-0.0526* (0.0292)	-0.3359 (0.3236)
	Bond	0.2675 (0.2106)	0.1995* (0.1009)	-3.4180*** (1.0405)
	Interest rate	-0.0204 (0.0141)	-0.0023 (0.0050)	0.3690*** (0.0680)
<i>6. Correlations/Volatilities</i>				
	State 1			
	Stock	0.0630*** (0.0050)		
	Bond	-0.1953*** (0.0020)	0.0172*** (0.0016)	
	Interest rate	0.0888** (0.0244)	-0.1603*** (0.0009)	0.2110*** (0.0098)
	State 2			
	Stock	0.0315*** (0.0014)		
	Bond	0.3039** (0.0238)	0.0114** (0.0100)	
	Interest rate	-0.1681** (0.0178)	-0.2905*** (0.0007)	0.1370*** (0.0077)
<i>7. Transition probabilities</i>				
	State 1			State 2
	State 1	0.8918 (0.4892)		0.1082
	State 2	0.0418 (0.4368)		0.9582
<i>8. State duration</i>				
	State 1		9.25 {0.2790}	
	State 2		23.90 {0.7210}	

Notes: The first panel refers to the single-state benchmark case ( $k = 1$ ). The VAR coefficient estimates should be read in the following way: the coefficient illustrates the impact of a change in the variable listed in the corresponding column on the variable listed in the corresponding row. Standard errors are reported in parentheses. \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

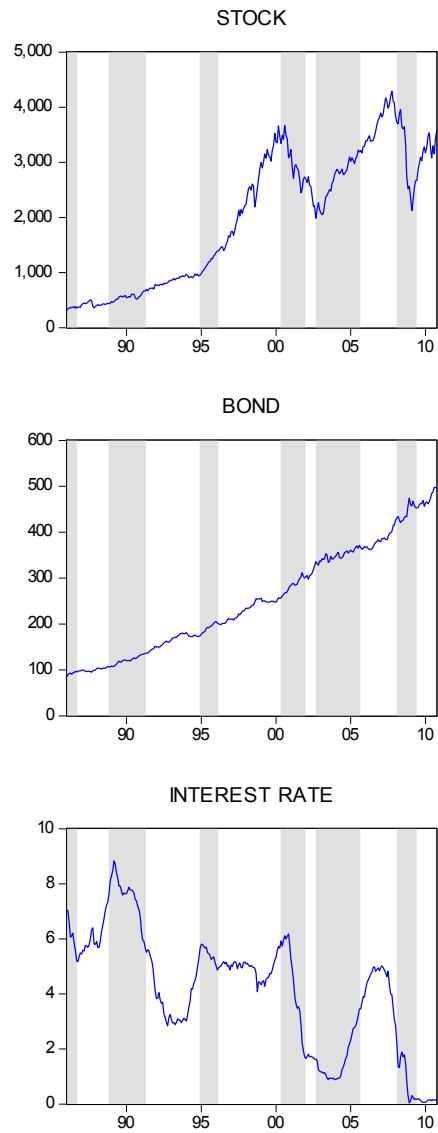
**Table 17**  
**Estimation Results of the Multivariate Regime Switching Model for UK Stock, Bond and Interest Rate Returns**

		Stock	Bond	Interest Rate
<b>A.</b>	<b>Model</b>	<b>VAR(1)</b>		
<i>1. Mean return</i>				
	<i>c</i>	0.0064** (0.0030)	0.0062*** (0.0011)	0.0078 (0.0169)
<i>2. VAR(1) matrix</i>				
	Stock	0.0827 (0.0588)	-0.0311 (0.0217)	1.0717*** (0.3314)
	Bond	0.1816 (0.1656)	0.1520** (0.0613)	-5.6707*** (0.9335)
	Interest rate	0.0078 (0.0089)	0.0026 (0.0033)	0.3944*** (0.0505)
<i>3. Correlations/Volatilities</i>				
	Stock	0.6651***		
	Bond	0.1596***	0.0913**	
	Interest rate	-0.0933**	-0.3384***	21.1175***
<b>B.</b>	<b>Model</b>	<b>MSIAH(2,1)</b>		
<i>4. Intercept</i>				
	$\mu_1$	0.0023 (0.0036)	0.0045*** (0.0012)	0.0014 (0.0076)
	$\mu_2$	0.0182*** (0.0060)	0.0105*** (0.0029)	0.0302 (0.0526)
<i>5. VAR(1) matrices</i>				
	State 1			
	Stock	-0.0002 (0.0802)	-0.0339 (0.0256)	0.506*** (0.1730)
	Bond	0.3180 (0.2202)	0.1814** (0.0721)	-1.4239** (0.4803)
	Interest rate	0.0063 (0.0250)	0.0073 (0.0081)	0.5047*** (0.0606)
	State 2			
	Stock	0.1961** (0.0876)	-0.0233 (0.0421)	2.0206** (0.8027)
	Bond	-0.1508 (0.2659)	0.0680 (0.1297)	-11.5543*** (2.4227)
	Interest rate	0.0065 (0.0101)	0.0016 (0.0049)	0.2884*** (0.0935)
<i>6. Correlations/Volatilities</i>				
	State 1			
	Stock	0.0467*** (0.0023)		
	Bond	-0.1109*** (0.0011)	0.0145*** (0.0008)	
	Interest rate	0.0875*** (0.0071)	-0.1915*** (0.0073)	0.0986*** (0.0055)
	State 2			
	Stock	0.0465*** (0.0038)		
	Bond	0.5250*** (0.0023)	0.0225*** (0.0015)	
	Interest rate	-0.2879** (0.0460)	-0.4864** (0.0433)	0.4330*** (0.030)
<i>7. Transition probabilities</i>				
		State 1		State 2
	State 1	0.9505 (0.4122)		0.0495
	State 2	0.1223 (0.4330)		0.8777
<i>8. State duration</i>				
	State 1		20.22 {0.7121}	
	State 2		8.18 {0.2879}	

Notes: The first panel refers to the single-state benchmark case ( $k = 1$ ). The VAR coefficient estimates should be read in the following way: the coefficient illustrates the impact of a change in the variable listed in the corresponding column on the variable listed in the corresponding row. Standard errors are reported in parentheses. \* denotes significance at 10%, \*\* significance at 5%, \*\*\* significance at 1%.

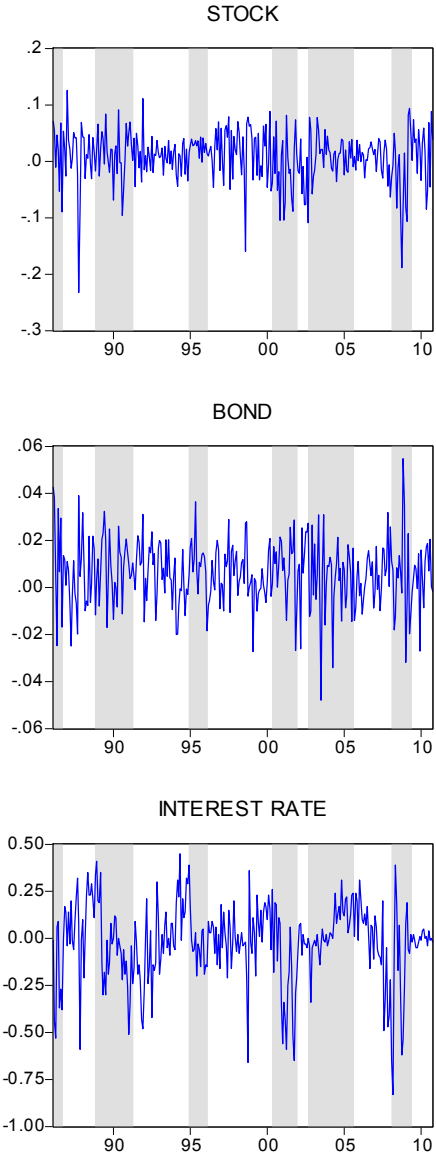
# APPENDIX B

Figure 1  
Total Stock Market Return Index, Total Bond Return Index and 3 Month Treasury Bill for  
United States

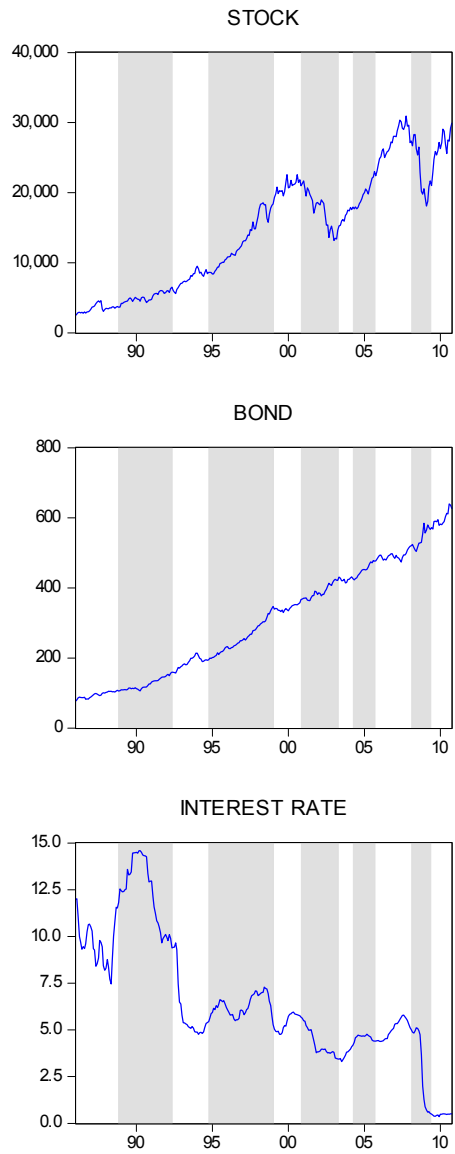




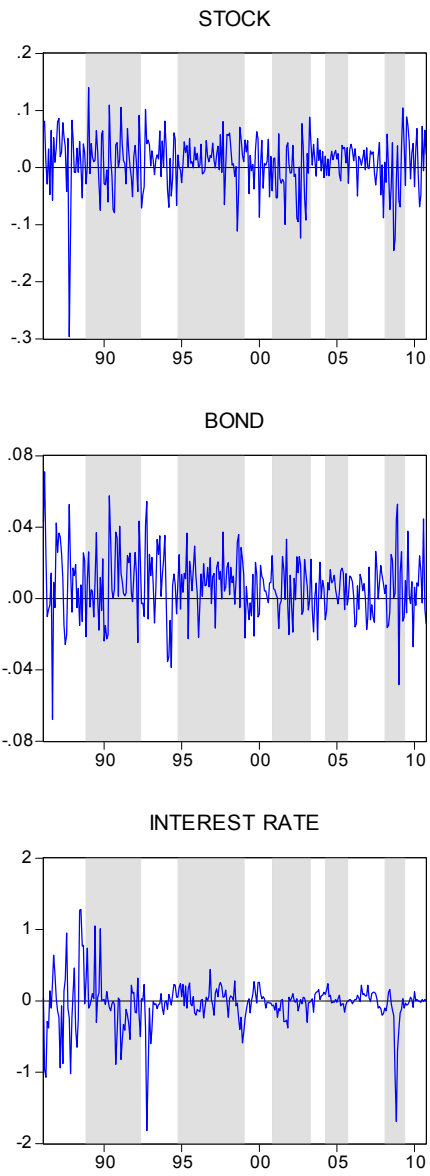
**Figure 2**  
**Return Series for Stock, Bond and Interest Rate for United States**



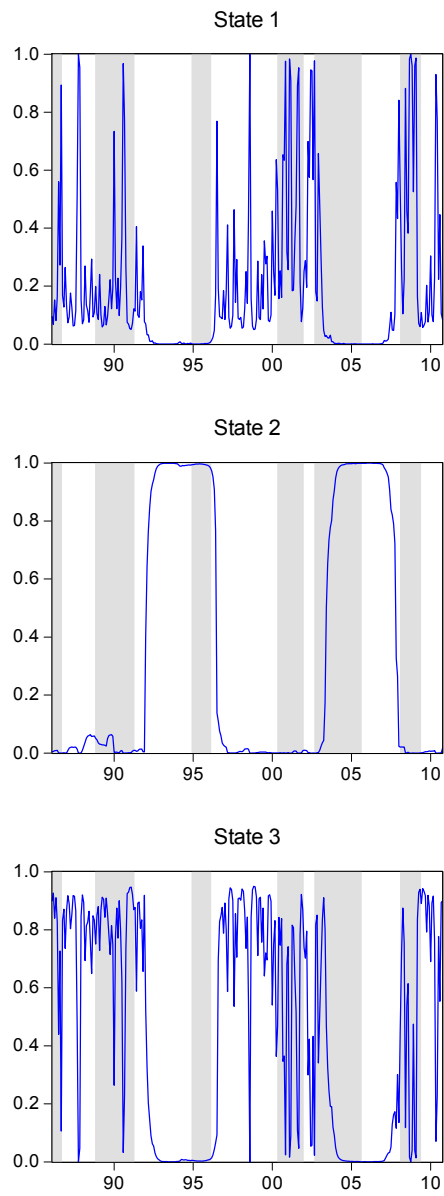
**Figure 3**  
**Total Stock Market Return Index, Total Bond Return Index and 3 Month Treasury Bill for United Kingdom**



**Figure 4**  
**Return Series for Stock, Bond and Interest Rate for United Kingdom**



**Figure 5**  
**Smoothed State Probabilities of the Univariate Model for US Stock Returns**



**Figure 6**  
**Smoothed State Probabilities of the Univariate Model for UK Stock Returns**

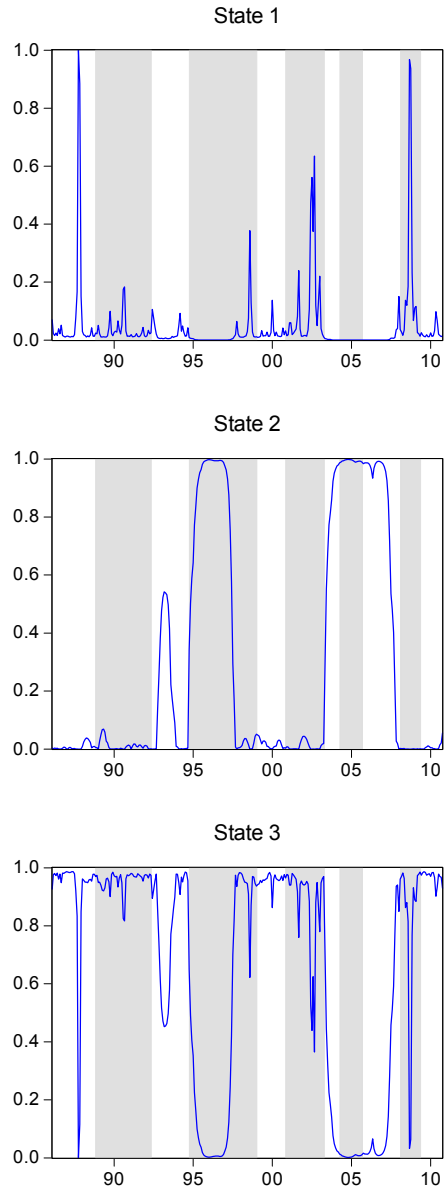
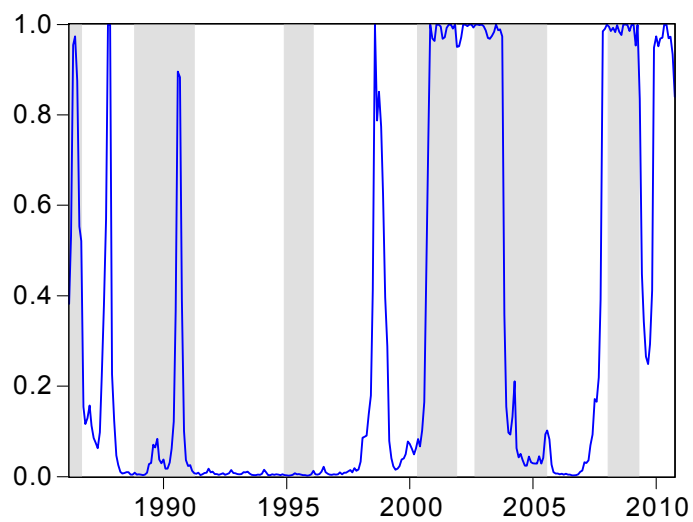


Figure 7  
Smoothed State Probabilities of the Multivariate Model for US Stock and Bond and Returns

State 1



State 2

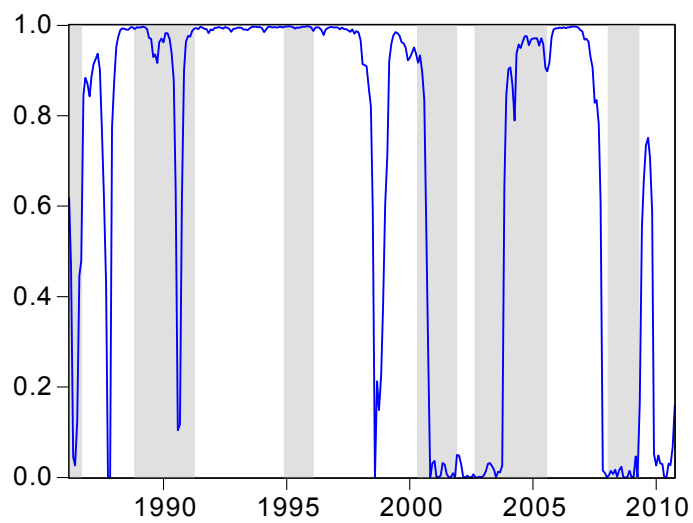


Figure 8  
Smoothed State Probabilities of the Multivariate Model for UK Stock and Bond and  
Returns

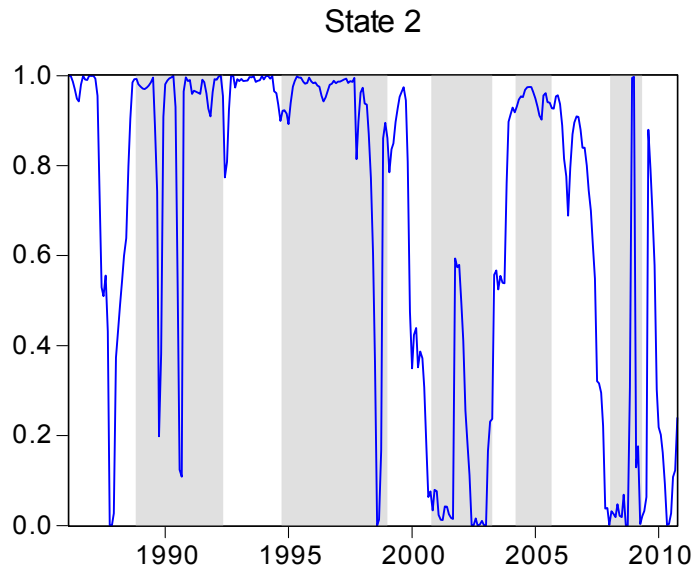
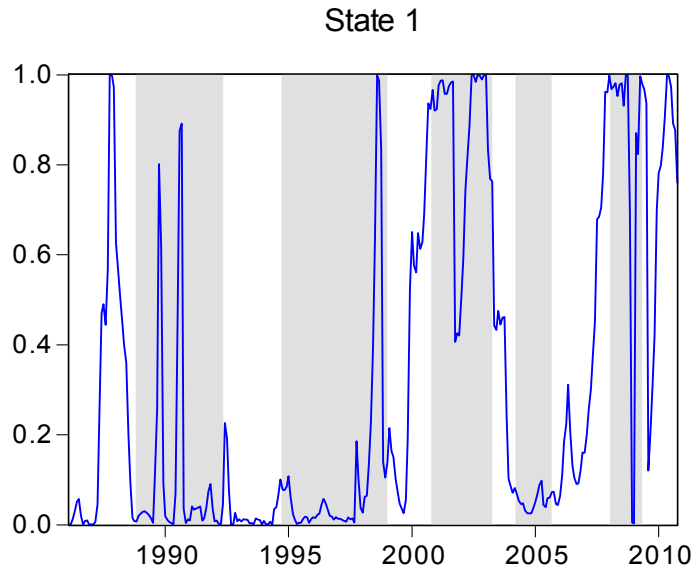
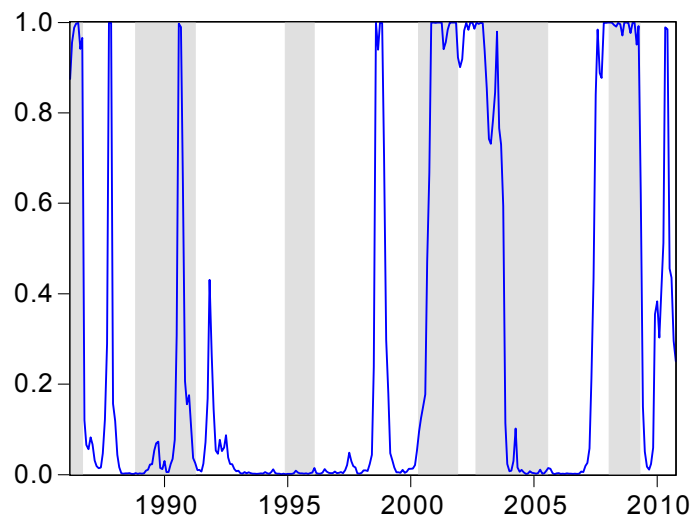


Figure 9  
Smoothed State Probabilities of the Multivariate Model for US Stock, Bond and Interest  
Rate Returns

State 1



State 2

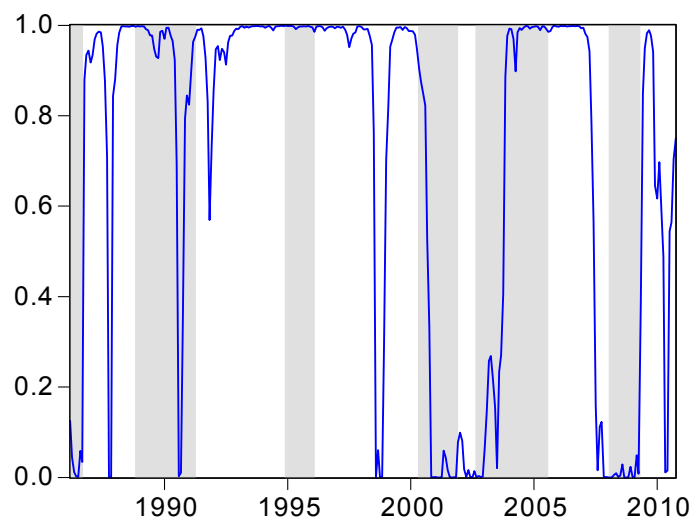




Figure 10  
Smoothed State Probabilities of the Multivariate Model for UK Stock, Bond and Interest  
Rate Returns

