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## A NEW REAL-TIME INDICATOR FOR THE EURO AREA GDP

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**ABBREVIATIONS:**

DFT – discrete Fourier transform  
 ECB –European Central Bank  
 EMSFE – estimated mean squared filter error  
 EU – European Union  
 I(i) – integrated of order i  
 GARCH – generalised autoregressive conditional heteroskedasticity  
 GDP – gross domestic product  
 MSFE – mean squared filter error  
 OECD –Organization for Economic Co-operation and Development  
 PMI – Purchasing Managers Index  
 SA – seasonal adjustment, seasonally adjusted  
 UK – the United Kingdom  
 US – the United States

**ABSTRACT**

The paper proposes a new real-time unrevised indicator tracking medium-to-long-term component in the quarterly growth of the euro area GDP. The new indicator is based on recently developed real-time filtration methodology, the multivariate direct filter approach, applied to selected business and consumer survey and share price data. The new indicator is found to have led another established indicator, the Eurocoin, by about three months since mid-2009 and be about coincident with but smoother than the PMI. In addition to the euro area aggregate indicator, the paper presents prototypical indicators for four biggest EU economies – Germany, France, the UK and Italy. Overall, the described filter approach appears to be able to provide somewhat better results in tracking business cycle developments than other widely used approaches.

**Keywords:** real-time signal extraction, coincident indicator, multivariate direct filter approach

**JEL code:** C13, C32, E32, E37

## INTRODUCTION

The paper applies a recently developed real-time filtration methodology, the multivariate direct filter approach (Wildi (2011)), to construct a new real-time unrevised indicator tracking medium-to-long-run component in the quarterly growth of euro area GDP.

The demand for real-time macroeconomic indicators exists mainly because many key macroeconomic variables are released with a considerable lag and are subsequently revised in later releases. For example, the first rough estimate of the main macroeconomic aggregate, GDP, called "flash GDP", is released only 45 days after the reference period in the EU, including the euro area, and happens to be revised substantially. The first official release of the EU GDP is published only 65 days after the reference period, and even this first release can be revised in subsequent releases. However, economic and financial agents are keen to have timely information on new developments in economy. Therefore, efforts are made to use timely information in order to capture the overall economic picture in a more timely manner. That is why several indicators are available that try to map timely disaggregate information on the aggregate series like the GDP. Particularly, there are several established real-time indicators tracking the economic situation in the euro area, a few of which are subjectively selected below.

The (New) Eurocoin (Altissimo et al. (2010)) is a replacement of its predecessor (Old) Eurocoin (Altissimo et al. (2001)) since May 2009. This indicator tracks medium-to-long-run component in the quarterly growth of euro area GDP and is a result of a dynamic factor model applied to approximately 145 series.

The Euro Growth Indicator by EUROFRAME<sup>1</sup> is a monthly nowcast and a forecast of the euro area GDP two quarters ahead of official statistics. It is based on a bridge regression applied to timely survey and financial data.

The new Now-Casting Economics Limited<sup>2</sup> project produces a nowcast of the quarterly growth of euro area GDP on a weekly basis. Its real-time performance currently spans only the last three quarters, hence it is premature to assess the quality of this indicator and, particularly, the value added from weekly, as opposed to monthly, updating frequency.

The OECD publishes a composite leading indicator for the euro area (Arnaud and Hong (2001)) which, at the time of writing this paper, is a revised monthly indicator targeting a lead in the business cycle (bandpass) of the euro area industrial production index. A drawback of the revised indicator, however, is that its revised values might send wrong signals to its users about its real-time performance. Another nuance is that a bandpass excludes trend growth, which plays an important role in the existence and depth of recessions.

A quarterly Ifo Economic Climate Indicator for euro area<sup>3</sup> is updated only quarterly, which might be less frequent than some economic agents would like.

<sup>1</sup> See [www.euroframe.org](http://www.euroframe.org).

<sup>2</sup> See [www.now-casting.com](http://www.now-casting.com).

<sup>3</sup> See <http://www.cesifo-group.de/portal/page/portal/ifoHome/a-winfo/d1index/25indexweseuro>.

The monthly Markit flash euro area PMI<sup>4</sup> is one of the closely watched indices for the euro area economy because it is timely, unrevised, has straightforward methodology and is economically relevant. It is based on surveys of companies.

This paper applies a recently developed multivariate real-time filtration technique to selected survey and stock price data to create a coincident, unrevised indicator for euro area GDP and compares its performance with the one of PMI and Eurocoin. Various robustness checks are performed, as well as prototypical indicators for the biggest four EU economies – Germany, France, the UK and Italy are also presented. The main finding is that the new indicator has been leading the Eurocoin by about three months since mid-2009 and is about zero to one month ahead of and smoother than the PMI. Overall, the described filtration methodology is found to provide somewhat better results in tracking business cycle developments than other widely used approaches.

The paper is structured as follows. Section 2 reviews the filtration methodology used in the construction of the new indicator. Section 3 applies the filtration methodology to construct the new indicator, performs real-time comparison with the Eurocoin and the PMI as well as constructs prototypical indicators for the four biggest EU economies. Section 4 concludes, while Appendix elaborates on the methodology in more detail.

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<sup>4</sup> See <http://www.markiteconomics.com/Survey/Page.mvc/PressReleases>.

## 1. FILTRATION METHODOLOGY

This paper is concerned with estimating a signal – a trendcycle or a business cycle – in real time. Let us denote  $y_T$  as the ideal output of a symmetric, possibly bi-infinite filter  $\sum_{j=-\infty}^{\infty} \gamma_j L^j$  applied to input series  $x_T$ :

$$y_T = \sum_{j=-\infty}^{\infty} \gamma_j L^j x_T = \sum_{j=-\infty}^{\infty} \gamma_j x_{T-j} \quad (1)$$

where  $L$  is the lag or backshift operator. The filter in equation (1) is called the ideal filter and the filter output  $y_T$  is called the ideal filter output. Usually, though, time series are finite in practice and, therefore the ideal filter is infeasible. A practitioner might use a finite symmetric filter as an approximation to equation (1) in the middle of the time series but even the symmetric approximation is infeasible at the very end of the sample, i.e. in real time. A real-time estimate of  $y_T$ , given a finite input  $\{x_1, \dots, x_T\}$ , can be written as

$$\hat{y}_T = \sum_{j=0}^{T-1} b_j x_{T-j} \quad (2).$$

It is a well-know fact that the filter in equation (2) generally possesses a nontrivial phase shift, i.e. its output is lagging in time. In order to define the phase shift and, in general, the effect of a filter, let us denote the generally complex transfer functions of filters in equations (1) and (2) by  $\Gamma(\omega) = \sum_{j=-\infty}^{\infty} \gamma_j \exp(-ij\omega)$  and  $\hat{\Gamma}(\omega) = \sum_{j=0}^{T-1} b_j \exp(-ij\omega)$  respectively. A generally complex number  $\Gamma(\omega)$  can be represented in polar coordinates as  $\Gamma(\omega) = A(\omega) \exp(-i\Phi(\omega))$ , where  $A(\omega) = |\Gamma(\omega)|$  is the amplitude, and  $\Phi(\omega) = -\arg(\Gamma(\omega))$  is the phase. The effect of a filter as represented by its transfer function can be summarised by the effect of amplitude and phase functions. The amplitude function  $A(\omega)$  can be interpreted as the weight, amplification or damping, attributed by the filter to the input signal at frequency  $\omega$ . The phase function  $\Phi(\omega)$  can be interpreted as a shift function of the amplified or damped signal at frequency  $\omega$ . This paper is concerned with obtaining a one-sided filter output, which would be a good, i.e. as good as it can be in real time, approximation to the ideal output. Let us consider stationary processes because this paper applies the filter to nonintegrated (I(0)) processes. Generalisation to integrated processes is straightforward by using pseudo spectral estimates and filter constraints at frequency zero; see Wildi (2008). For a stationary process  $x_T$  the mean squared filter error (MSFE) can be expressed as the mean squared difference between the ideal output and the real-time estimate:

$$\int_{-\pi}^{\pi} |\Gamma(\omega) - \hat{\Gamma}(\omega)|^2 dH(\omega) = E[(y_T - \hat{y}_T)^2] \quad (3)$$

where  $H(\omega)$  is the unknown spectral distribution of  $x_T$ . A finite sample approximation of MSFE, (3), is:



$$\frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k |\Gamma(\omega_k) - \hat{\Gamma}(\omega_k)|^2 S(\omega_k) \quad (4)$$

where  $\omega_k = k2\pi/T$ ,  $[T/2]$  is the greatest integer smaller or equal to  $T/2$ , and the weight  $w_k$  is defined as

$$w_k = \begin{cases} 1 & \text{for } |k| \neq T/2 \\ 1/2 & \text{otherwise} \end{cases} \quad (5)$$

(see Brockwell and Davis (1987), Chapter 10 for the reason for  $w_k$ ; without it the inverse DFT does not replicate the data perfectly).  $S(\omega_k)$  in expression (4) can be interpreted as an estimate of unknown spectral density of  $x_T$ , which can be any spectral estimate, e.g. the one of white noise (Baxter and King (1999)), random walk (Christiano and Fitzgerald (2003)), an ARIMA-based spectral estimate as used in the TRAMO/SEATS<sup>5</sup> seasonal adjustment procedure (Caporello, Maravall and Sanchez (2001)), or a particular ARIMA(0,2,2) implicitly assumed by the Hodrick-Prescott filter (Hodrick and Prescott (1997)). However, as discussed in Wildi (2008), consistency of  $S(\omega_k)$  is not required because the goal is not to estimate  $dH(\omega)$  but the filter mean squared error (see equation (3)). Therefore, this paper uses a "sufficient statistic", i.e., a periodogram  $I_{Tx}(\omega_k)$ , as  $S(\omega_k)$  in expression (4):

$$S(\omega_k) := I_{Tx}(\omega_k) = \frac{1}{2\pi T} \left| \sum_{t=1}^T x_t \exp(-it\omega_k) \right|^2 \quad (6).$$

Minimising expression (4) yields the real-time filter output optimally approximated to the ideal output in the sense of mean squared error problem. As noted above, expression (4) is a generalised problem that encompasses the problem of Baxter and King (1999) (where the feasible filter is assumed to be a symmetric bandpass and the spectral estimate  $S(\omega_k)$  is the spectrum of white noise), Christiano and Fitzgerald (2003) (where the feasible filter is assumed to be a bandpass and the spectral estimate  $S(\omega_k)$  is the spectrum of random walk), TRAMO/SEATS seasonal adjustment methodology using the Wiener-Kolmogorov filter (ARIMA-based spectral estimates), the Hodrick-Prescott filter (which can be interpreted to yield optimal results for a specific ARIMA(0,2,2) input process and a particular filter amplitude; see Maravall and R  o (2001), and King and Rebelo (1993)), etc. Yet, Wildi (2008) proposes a customised version of expression (4), which in this paper is found to be useful in creating a real-time indicator, and which is described in the next subsection.

<sup>5</sup> See <http://www.bde.es/webbde/es/secciones/servicio/software/econom.html>.

### 1.1 Univariate direct filter approach

Wildi (2008) proposes a customised version of expression (4), a part of which is implemented in this paper. We rewrite discrete version MSFE (expression (4)) as

$$\frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k |\Gamma(\omega_k) - \hat{\Gamma}(\omega_k)|^2 I_{Tx}(\omega_k) W(\omega_k) \quad (7),$$

which is identical to expression (4), for  $W(\omega_k) := 1$ . However, a more general version of  $W(\omega_k) := W(\omega_k, expw, cut)$  can be written:

$$W(\omega_k, expw, cut) = \begin{cases} 1 & \text{if } |\omega_k| < cut \\ (1 + |\omega_k| - cut)^{expw} & \text{if } |\omega_k| \geq cut \end{cases} \quad (8),$$

which collapses to unity for  $expw = 0$ , in which case the classical mean squared optimisation (expression (4)), is obtained. Parameter *cut* (for "cut-off frequency") marks the transition between passband and rightmost stopband, and positive values of *expw* (for "exponent weight") emphasise high-frequency components in the rightmost stopband, thus, making the filter output smoother than the one obtained by minimising expression (4) for positive *expw*. The derivation of the filter based on minimisation of expression (7) and (8) is described in Appendix.

Univariate filters can be useful in case the target variable of interest  $x_t$  is timely; however, it is usually not the case for macroeconomic series. For instance, one of the key macroeconomic variables GDP is released with a delay and is revised in subsequent releases. Therefore, a practitioner might be interested in using other economic variables with shorter delays in creating benchmark indicators for GDP of some release. This environment demands the ability for a multiple-series analysis, which in the particular context relates to the next subsection.

### 1.2 Multivariate direct filter approach

The above univariate customised filter has been generalised to a multivariate filter in Wildi (2011). Rewriting of the univariate minimisation problem in expression (7) with the discrete Fourier transform (DFT)  $\Xi_{Tx}(\omega_k)$  gives:

$$\begin{aligned} & \frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k |\Gamma(\omega_k) - \hat{\Gamma}(\omega_k)|^2 I_{Tx}(\omega_k) W(\omega_k) \\ &= \frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k |\Gamma(\omega_k) \Xi_{Tx}(\omega_k) - \hat{\Gamma}(\omega_k) \Xi_{Tx}(\omega_k)|^2 W(\omega_k) \end{aligned} \quad (9)$$

where

$$\Xi_{Tx}(\omega_k) = \sqrt{\frac{1}{2\pi T}} \sum_{t=1}^T x_t \exp(-it\omega_k) \quad (10).$$



In addition to the filter output  $y_T$  and the corresponding input  $x_T$  let us assume that there are  $m$  additional explanatory variables  $z_{jT}$ ,  $j=1, \dots, m$  that might help improve the real-time estimate of  $y_T$  obtained with a univariate filter. Then, the second expression in the modulus on the second line of expression (9),  $\hat{\Gamma}_X(\omega_k)\Xi_{Tx}(\omega_k)$ , becomes

$$\hat{\Gamma}_X(\omega_k)\Xi_{Tx}(\omega_k) + \sum_{n=1}^m \hat{\Gamma}_{z_n}(\omega_k)\Xi_{Tz_n}(\omega_k) \quad (11)$$

where

$$\hat{\Gamma}_X(\omega_k) = \left( \sum_{j=0}^L b_{xj} \exp(-ij\omega_k) \right) \Xi_{Tx}(\omega_k) \quad (12)$$

$$\hat{\Gamma}_{z_n}(\omega_k) = \left( \sum_{j=0}^L b_{z_n j} \exp(-ij\omega_k) \right) \Xi_{Tz_n}(\omega_k) \quad (13)$$

are the one-sided transfer functions applied to the explanatory variables, and  $\Xi_{Tx}(\omega_k)$ ,  $\Xi_{Tz_n}(\omega_k)$  are the corresponding DFTs. Then, the multivariate version of expression (9) can be written as

$$\frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k \left| \left( \Gamma(\omega_k) - \hat{\Gamma}_X(\omega_k) \right) \Xi_{Tx}(\omega_k) - \sum_{n=1}^m \hat{\Gamma}_{z_n}(\omega_k) \Xi_{Tz_n}(\omega_k) \right|^2 W(\omega_k) \quad (14).$$

The paper uses the filter obtained by minimising expression (14) subject to filter parameters and the first order constraint at frequency zero. See Appendix for further elaboration.

## 2. REAL-TIME INDICATOR FOR THE EURO AREA GDP

### 2.1 Data

#### The target

The target is medium-to-long-run fluctuations in quarterly growth of seasonally adjusted euro area GDP in chain-linked prices, as published by Eurostat. Specifically, the filter is set to target an ideal lowpass of quarterly growth of seasonally adjusted euro area GDP with cut-off wave length of 12 months (see Figure 5 in Appendix). The usually defined minimum length of a business cycle is 18 months (Burns and Mitchell (1946), Baxter and King (1999), Christiano and Fitzgerald (2003)). However, passing higher-frequency content through a filter allows for both faster turning point detection and closer amplitude fit. The quarterly GDP data are taken from Q1 1995 onwards. The data are linearly interpolated to monthly frequency, logged, regularly differenced and demeaned before their spectral content enters the filter. Alternatively, interpolation and differencing can be performed in the frequency domain but the results show no obvious improvement of the latter compared to such data transformation in the time domain.

### Explanatory variables

Select monthly confidence indicators published by DG Ecfm and the US share price index published by Eurostat are used as explanatory variables for the euro area GDP. DG Ecfm data are usually published at the end of reference month, except for December for which data are published in early January. Ecfm business and consumer survey data are almost unrevised; this applies to both seasonally unadjusted and seasonally adjusted data, as the latter is the product of a seasonal adjustment program "Dainties" that does not revise history as new data come in.<sup>6</sup> The above-mentioned considerations make DG Ecfm data comfortable for real-time filtration. The selected business and consumer confidence data are the production trend observed in recent months (industry), the assessment of order-book levels (industry), the assessment of stocks of finished products (industry), production expectations for months ahead (industry), employment expectations for months ahead (industry), the confidence indicator in construction, the confidence indicator in retail trade, and the consumer confidence indicator. The choice of indicators is based on economic relevance and data availability.

Other Ecfm survey data have been tested and found not to add substantial quality to the indicator. Seasonally adjusted Ecfm data are used (see Figure 4 in Appendix).

The monthly US share price index as published by Eurostat is used as an additional explanatory variable due to potential economic spillovers from the US. It is published approximately within one week after the reference month. Since the real-time indicator as presented in this paper is set to be published at the end of reference month, the US share price index enters the filter with a lag of one month. Thus, there is a room for potential improvement in this aspect, as financial data generally are available on the go.

All explanatory variables are taken from January 1995 onwards, regularly differenced and standardised to zero mean and unit variance.

## 2.2 Filter specification

### Filter dimension

The filter is tested for various dimension sizes. Estimated mean squared filter error (EMSFE) given in expression (14), is the implemented statistic for goodness of fit, according to which low-dimensional filters often match the target DFT the best. 1-variable and 2-variable filters show about the same EMSFE, while 3-variable filter and 4-variable filter EMSFEs are about 3 and 9 times higher respectively. Thus, 2-variable filter is chosen. Although the GDP series itself could be filtered, it is chosen not to do so as the GDP series is substantially lagged and revised, and appears not to be useful in prompt real-time filtration. Therefore, only the spectral content of the GDP series is fed to the filter, and the monthly variables are set to target that spectral content. Thus, a 2-variable filter means that two monthly variables are filtered to target the latent output of the ideal filter that would be applied to the GDP series.

<sup>6</sup> For details, see *The joint harmonized EU programme of business and consumer surveys*, User Guide, 2007, European Commission Directorate-General for economic and financial affairs, available at [http://ec.europa.eu/economy\\_finance/db\\_indicators/surveys/documents/userguide\\_en.pdf](http://ec.europa.eu/economy_finance/db_indicators/surveys/documents/userguide_en.pdf).

The filter length is set to depend on the number of input variables and the sample size to control for overparameterisation. The particular 2-dimensional filter is 38 to 48 observations long, depending on the in-sample length.

### **Filter constraint**

The first-order integration constraint is imposed by setting the filter amplitude to unity at frequency zero. Since all the input series are demeaned and are  $I(0)$ , the first order integration constraint ensures tracking the scale of filter output close to that of target signal.

### **Noise suppression**

Extra suppression of high-frequency content in the stopband is implemented with positive *expw* parameter as described in the subsection 1.1. As a result, the filter output is smoother than the one of the classical mean squared filter error problem.

### **Cross-sectional aggregation**

Given that the considered set of explanatory variables contains more variables than the selected filter dimension, all possible combinations of variables are entered in the filter so that the final filter output is a combined 2-variable filter output weighted by weights proportional to inverse EMSFE. Alternatively, principal components (Stock and Watson (1998)) could be used to shrink the dimension of the data set to a few factors but the results show no obvious improvement in this regard.

### **Output re-scaling and adding the mean back**

The filter output obtained thus far is then rescaled to the variance of the output of 31-month-long finite symmetric filter for the time span available at the particular real-time estimation moment. Although the first order filter restriction ensures to some extent that the scale of the filter output be the same as the target signal, such rescaling, nonetheless, is harmless and is proved to be useful. Finally, the mean of the output of finite symmetric filter is added to the filter output. One could use the mean of GDP series instead, but since the latter is more volatile in a real-time setting than the output of finite symmetric filter, the latter is preferred.

### **Averaging along time dimension**

At this point, one can produce the real-time indicator. However, a couple of issues emerge. If filter coefficients are updated every time new data become available but the historic values of the indicator are not updated, the noise generated by the changing estimated latent level of the target signal can suppress the estimated signal. Figure 6 in Appendix shows an indicator resulting from such filter coefficient re-estimation every quarter, i.e. the noise induced by frequent coefficient updates suppresses the estimated target signal. In this light, one might choose an in-sample span yielding most satisfactory out-of-sample performance and fix the coefficients. In this case, the real-time estimate is smooth. A real-time recession indicator for the US economy, the USRI<sup>7</sup>, is based on the latter approach. Yet, another issue emerges:

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<sup>7</sup> See <http://www.idp.zhaw.ch/usri>.

sooner or later the filter coefficients might need an update due to possible structural changes in the economy.

The above issues are solved with the following filter averaging along the time dimension. Filter coefficients are updated every time new GDP data become available. The final indicator is an average of all filter outputs obtained up to the estimation period. Furthermore, due to the revisions of GDP series, some of its observations at the end of series, denoted as "gap", are left out of the sample. In other words, the whole sample length is denoted as "len". The maximum in-sample period spans observations  $1:len - gap$ . Filter coefficients for reduced in-sample length  $1:len - gap - 3$ ,  $1:len - gap - 6$ , etc. are iteratively estimated until some minimum in-sample length, but the filter output is always estimated for the whole sample span, "len". Then an arithmetic average<sup>8</sup> of all historically estimated indicators is taken. More indicators are averaged as more data come in but the maximum aggregation time span is set to five years in order to take into account possible structural changes. This updating mechanism overcomes the problem induced by the level of target series and at the same time robustifies signal detection and incorporates new information/possible structural changes as new data become available. Setting *gap* to 9 or 15 months does not make much difference but insures against GDP revisions at the end of series and ensures that the target is about the "final" GDP as opposed to a "first release". The presented indicator is a result of setting the "gap" to nine months.

Such mechanism is in use for the last ten years of the available data. The out-of-sample time span is limited by the filter and sample lengths. The resulting filter output is shown in Figure 1, along with quarterly growth of euro area GDP and output of a 31-months long finite symmetric trendcycle filter.

*Figure 1*  
**The new real-time indicator**

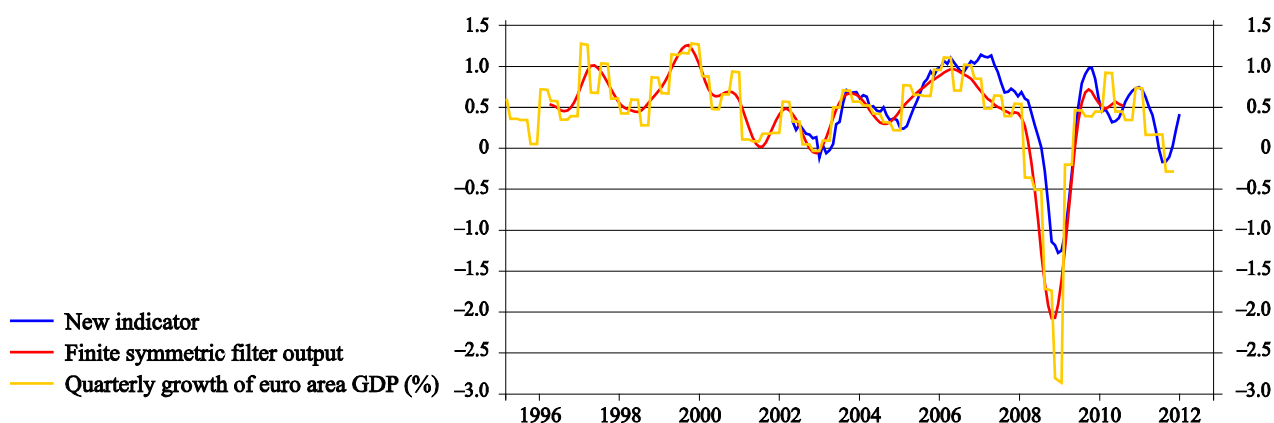


Figure 1 shows that the new indicator is slightly lagging in a historic perspective, and that its amplitude is comparable to that of the output of finite symmetric filter except for the great recession episode. A visual observation suggests that the indicator gets slightly faster after the great recession episode, when it appears to be

<sup>8</sup> A weighted average proportional to inverse EMSFE was also tested and found not to yield superior results.

coincident with the GDP growth. Methodologically, the indicator is constructed as a coincident indicator. Furthermore, the indicator gets slightly smoother over time. The improvement in both the speed and smoothness over time could be explained by the increasing data length available to the filter. Moreover, the smoothness performance is partly due to the filter averaging along the time dimension: the first observation of indicator in early 2002 is a result of single cross-sectionally aggregated filter output; the next indicator values are averages of increasing number of filter outputs along the time dimension until five years pass, when only the filter outputs over the last five years are averaged to account for the effect of different phases of business cycle and possible structural changes on the indicator's performance.

## 2.3 Robustness checks

### Adjustment for changing volatility

The implemented filter, like its special versions, e.g. Baxter-King (Baxter and King (1999)), Christiano-Fitzgerald (Christiano and Fitzgerald (2003)), Hodrick-Prescott (Hodrick and Prescott (1997)), Wiener-Kolmogorov (Kolmogorov (1941), Wiener (1949)), assumes inputs with constant volatility. However, economic data might be subject to changing volatility. Filter assuming inputs with constant volatility but applied to inputs with changing volatility might yield spurious output.

In this light, GARCH(1,1) (Bollerslev (1986); see Appendix for more details) is applied to survey data as a simple check for changing volatility. Indeed, survey data are found to be subject to heteroskedasticity (see Figure 8 in Appendix for estimated changing standard deviations). Therefore, GARCH(1,1)-normalised data are fed to the filter. However, GARCH-augmented filter slightly loses to non-GARCH one with respect to the estimated filter error and the amplitude fit during the great recession episode. Thus, GARCH normalisation of input data is not implemented in this paper, while the issue of the effect of heteroskedasticity on the filter output remains.

### Other data and transformations

Other additional data (e.g. other DG Ecfm data, new registered cars, industrial production, producer/consumer price indices, more financial variables) were tested and found not to yield conceptually different results, therefore, their usage is under consideration but excluded from the paper for simplicity and the lack of proper vintage data.

Both seasonally adjusted and unadjusted survey data were tested as inputs. Moreover, seasonal adjustment that excluded a stable seasonal component was tested. Preliminary results have shown that the use of seasonally adjusted survey data by Dainties for filter inputs gives the most satisfactory results. Another nuance is whether to use seasonally adjusted or unadjusted GDP for spectral estimates. If the target is lowpass with threshold to the left of  $\pi/6$ , both could be used without much difference in the outcome. However, since the threshold is set to seasonal frequency  $\pi/6$ , seasonally adjusted GDP is used for the target spectral estimate.

### First-order constraint

The above specification implements the first-order integration constraint that ensures that the scale of output is compatible with the scale of target. This is because all series are demeaned, so the  $I(1)$  constraint tackles scale/variance fitting. An alternative, i.e. no-integration specification was tested as well. Although the alternative works well in sample, it yields inferior real-time performance with respect to scale fitting compared to the  $I(1)$  specification. Specifically, the amplitude fit during the great recession episode is clearly worse than that for the  $I(1)$ -constrained filter.

### Data revisions

The SA method Dainties applied to Ecfm survey data does not induce revisions, as opposed to the results of the most popular SA methods X12<sup>9</sup> and TRAMO/SEATS. However, the last month of non-adjusted data might be slightly revised thus inducing revisions also in SA data. These revisions have magnitude of about 0.1 in absolute value for unadjusted data, if they occur (some series have been found not to be revised). The largest revisions are observed for the "construction confidence indicator" series and are of about 0.5 in absolute value. DG Ecfm does not provide vintage data publicly, however, DG Ecfm was very kind to provide vintages of non-adjusted data upon request. The results show that the output of the implemented filtering technique proved to be robust against these minor revisions.

As another vintage data source, the real time data base<sup>10</sup> maintained by the ECB provides vintage data for six out of eight seasonally adjusted variables used in this paper until February 2011. Unfortunately, the provided data are rounded to zero digits after comma. Nonetheless, the comparison of first release vintages with the latest ones results in the estimated mean deviation being zero, mean absolute deviation being about 1/4, and maximum absolute deviation ranging from 0.5 to 0.7, which basically reflects the effects of rounding. The filter output from the first release inputs is indistinguishable with a naked eye from the one with final release data, thus it is not presented.

Another source of potential data revision is revisions in GDP data. However, the solution to a revised GDP series is straightforward: the GDP series itself is not filtered; instead, only its spectral content is fed to the filter, therefore, a robust filter output is obtained by feeding the filter with estimated spectra of the final/revised GDP by dropping several observations from the end of the series. It means that the described method does not use uncertain first releases, and the dropping of last 3 or 5 quarters from the estimation routine gives similar results. The results described in the paper are obtained by dropping last 3 quarters of data.

Therefore, the plotted indicator's performance is expected not to deteriorate with new real-time data observations entering the estimation routine.

<sup>9</sup> See <http://www.census.gov/srd/www/x12a/>.

<sup>10</sup> See <http://sdw.ecb.europa.eu/browse.do?node=4843526>.



### Filter updating frequency

The default filter updating period is set to every quarter, as new GDP data become available. Filter updating is performed in order to take into account potential structural changes. Moreover, the indicator has been checked for a different and less frequent filter updating frequency and is found to be robust, which is an expected result, since the spectral content of GDP data in the relevant passband is not expected to change rapidly.

### Passband specification

The defined passband is lowpass with a cut-off frequency  $\pi/6$  which corresponds to yearly frequency. Setting the cut-off to commonly defined upper bound of business cycle frequency of 18 months makes the indicator slightly slower, and the amplitude fit slightly worse. Therefore, keeping the cut-off low allows for higher-frequency content passing through that speeds up the turning point detection and improves the amplitude fit.

### The effect of high-frequency noise suppression

Figure 7 in Appendix shows the effect of high-frequency noise suppression in the rightmost stopband by the customised filter. The blue line corresponds to the classical mean squared optimisation problem with  $W(\omega_k) = 1$ , i.e. there is no extra noise suppression. The other two lines correspond to the customised filter output with  $expw = 0.5$  and  $expw = 1$  respectively. It can be seen that the indicator without extra noise suppression is the fastest and also the noisiest of the three. A user of real-time indicator might want to trade speed for extra smoothness so that the real-time signal would be more reliable. In that case, the filter customisation kicks in by suppressing high-frequency content in the stopband. Ideally, this extra noise suppression would not alter the phase function in the passband but in practice it does so to some extent, i.e. extra noise suppression in the stopband slows down filter output somewhat. The presented final indicator is the middle case with  $expw = 0.5$ .

## 2.4 Comparison with Eurocoin and Markit euro area PMI

### Eurocoin

Eurocoin (Altissimo et al. (2010)) is an established coincident indicator for euro area GDP. According to Banca d'Italia's website<sup>11</sup>, it is a smooth real time estimate of the quarterly growth of euro area GDP released at the end of reference month, and has not been revised since May 2009. It is a product of generalised dynamic factor model (Forni et al. (2000), Forni and Lippi (2001)), which according to Altissimo et al. (2010) is applied to about 145 series. By contrast, the new indicator is a result of a multivariate filter applied only to selected nine explanatory variables. Eurocoin and the new indicator have a few features in common as well, e.g. both target trendcycle in the quarterly growth of euro area GDP as defined by a lowpass with cut-off frequency  $\pi/6$ , and both are designed to be coincident indicators. As such, Eurocoin is a decent benchmark for the new indicator.

<sup>11</sup> See <http://eurocoin.bancaditalia.it/>.

Figure 2 shows Eurocoin, latest vintage of quarterly growth of euro area GDP, and the new indicator for the last ten years.

Figure 2

**The new real-time indicator and Eurocoin**

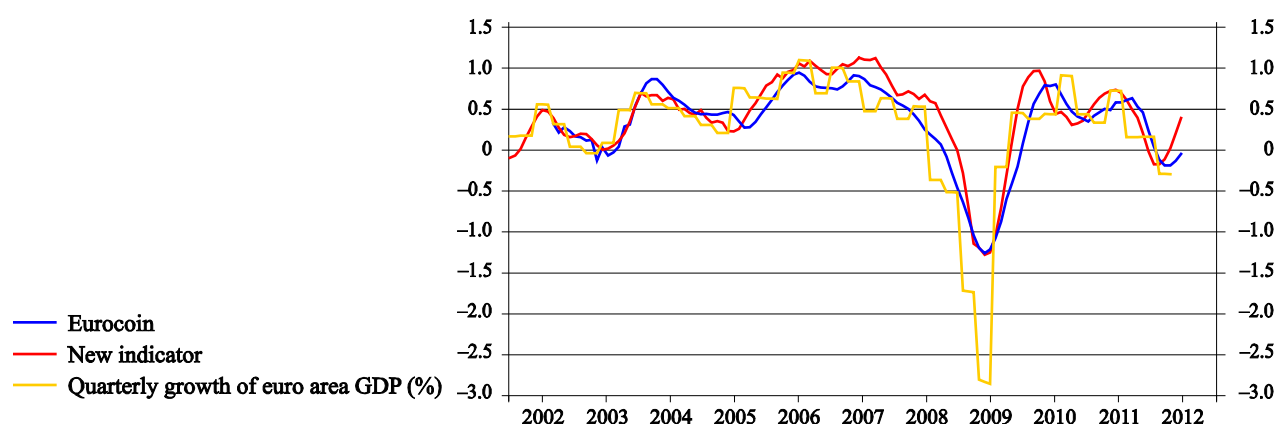
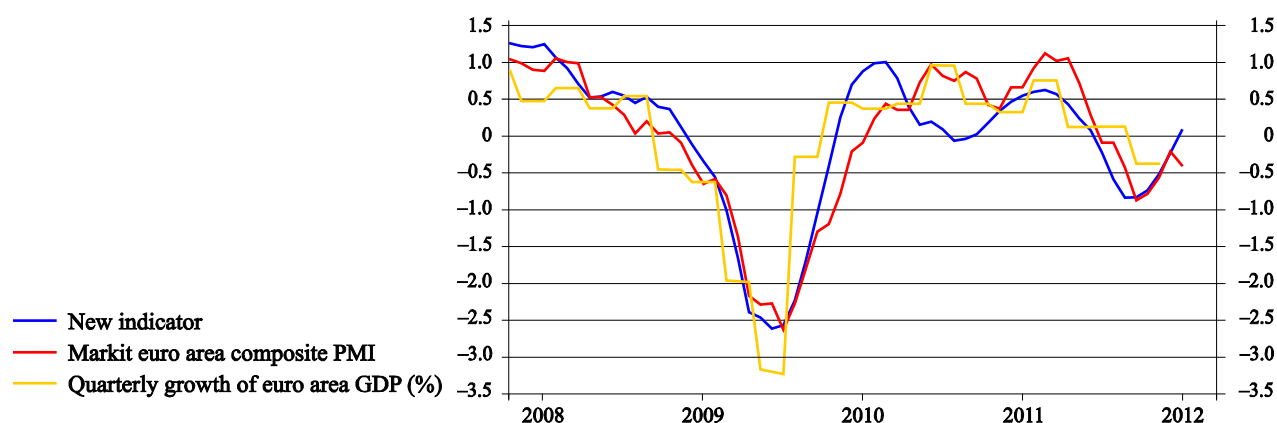


Figure 2 shows that the amplitudes of both indicators are similar over the considered period. The new indicator is about coincident with Eurocoin during 2002–2004 but less smooth due to the small in-sample period. The new indicator appears to be faster than Eurocoin during 2005 and coincident but with a worse level fit during 2007–2008. Eurocoin gets behind on average by about 3 months since 2009. It should be recalled that Figure 1 shows the new indicator to be slightly lagging behind the output of a finite symmetric filter for the whole episode of 2002–2008 and has become about coincident since 2009. Therefore, Figure 2 indicates that Eurocoin is lagging with respect to the output of a finite symmetric filter for the whole reference period 2002–2012. Given that the true out-of-sample period for Eurocoin begins only with May 2009, and that its pseudo real-time values are calculated using the last vintage data for the period before May 2009<sup>12</sup>, the real-time performance of Eurocoin may be assumed to be slightly worse than its pseudo real-time estimates suggest and that the new indicator potentially somewhat leads Eurocoin by a couple of months during true real time performance of both indicators.

### Markit euro area PMI

The Markit euro area PMI is revealed a few days after the end of reference month and is advertised as being closely correlated with the quarterly growth of euro area GDP. The PMI is one of the indices closely watched by economic and financial agents due to its early release, simple design and economic relevance. The PMI data for the last five years are collected from Bloomberg and plotted against the new indicator. Since the PMI is of different magnitude than the GDP growth, all variables are normalised to zero mean and unit variance for an easier comparison. Figure 3 shows that the new indicator is about coincident with the PMI while having one month lead during the troughs of the last two recessions, and being smoother.

<sup>12</sup> See the note in [http://eurocoin.cepr.org/files/file/Ecoin\\_realtime\\_99Feb12.xls](http://eurocoin.cepr.org/files/file/Ecoin_realtime_99Feb12.xls).

*Figure 3***The new real-time indicator and Markit euro area composite PMI.**

Note. All series are normalised to zero mean and unit variance.

## 2.5 Additional robustness check: indicators for four biggest EU economies in GDP terms

Figures 10–13 in Appendix show prototypical indicators for the German, French, UK and Italian GDPs whose designs are a copy/paste design from the euro area indicator with slight idiosyncratic modifications for each individual economy with respect to selected input data. The precise list of input data for each country is given in Appendix. Figures 10–13 show that the filter design works quite well for the German and French GDPs, slightly less so for the Italian GDP, and its quality is even worse for the UK. These results show that the filter design has some merits, yet every region has its own specificities to be accounted for by differentiating the input data.

## CONCLUSIONS

The paper applies a recently developed filtration methodology, i.e. the multivariate direct filter approach, to selected business and consumer confidence indicators and share price data to construct a real-time indicator tracking medium-to-long-run component of quarterly growth of euro area GDP.

The results show that the new indicator behaves similarly to another established indicator, Eurocoin, but leads it by about 3 months after mid 2009. Since the new indicator is designed as a coincident indicator, the results suggest that Eurocoin might be slightly lagging. The new indicator is also compared to the Markit euro area composite PMI and is found to be about zero to one month ahead, and smoother as well.

Various robustness checks which suggest that the indicator is quite robust have been performed. Prototypical indicators for four biggest EU economies are also presented, which show that the indicator design works quite well for Germany and France, less so for Italy and even worse for the UK, indicating that every region has its own specificities to be accounted for by selecting proper explanatory variables, but, overall, that the indicator design has its merits.

## APPENDIX

Figure 4  
DG Ecfm survey data

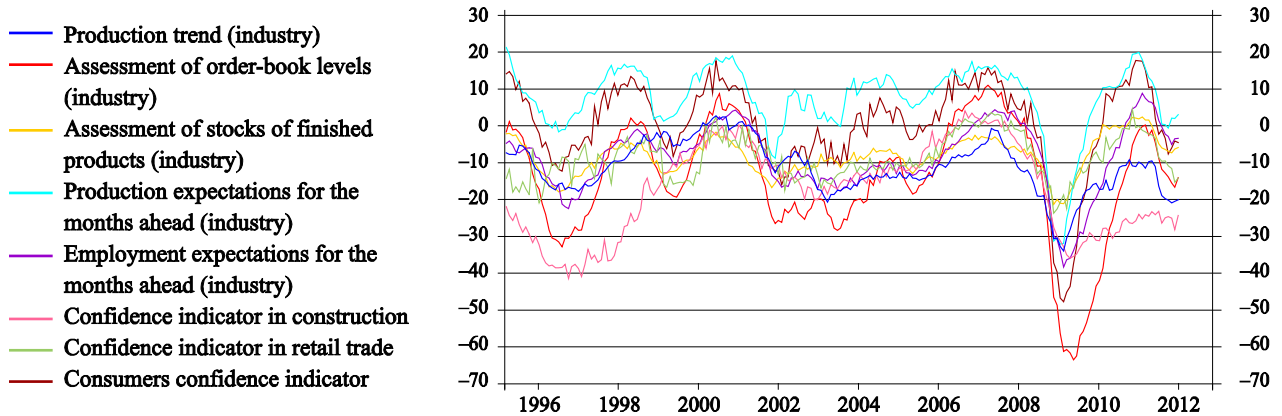
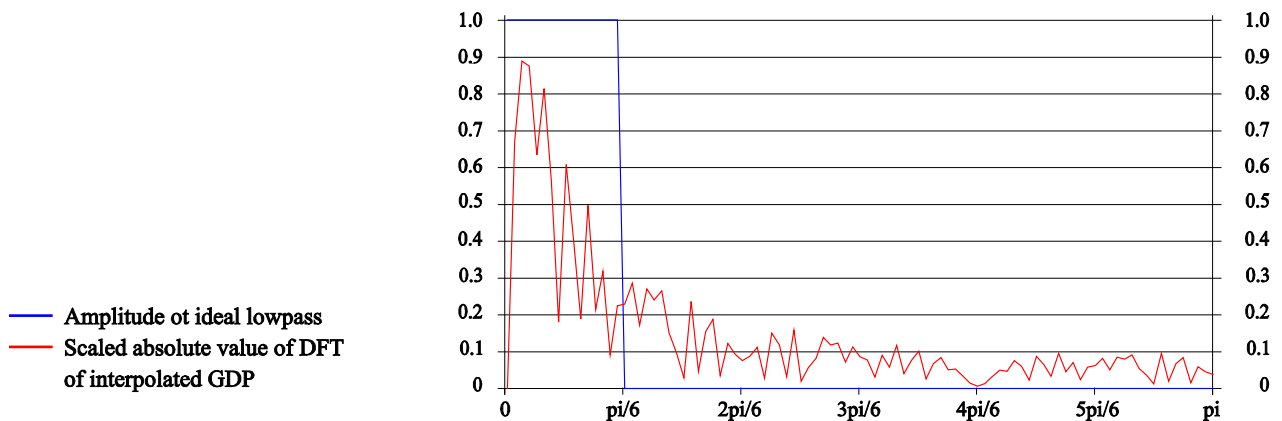


Figure 5  
Filter target



Note: The latter is zero in frequency zero because GDP series is demeaned.

Figure 6  
Filter output resulting from frequent filter coefficient update

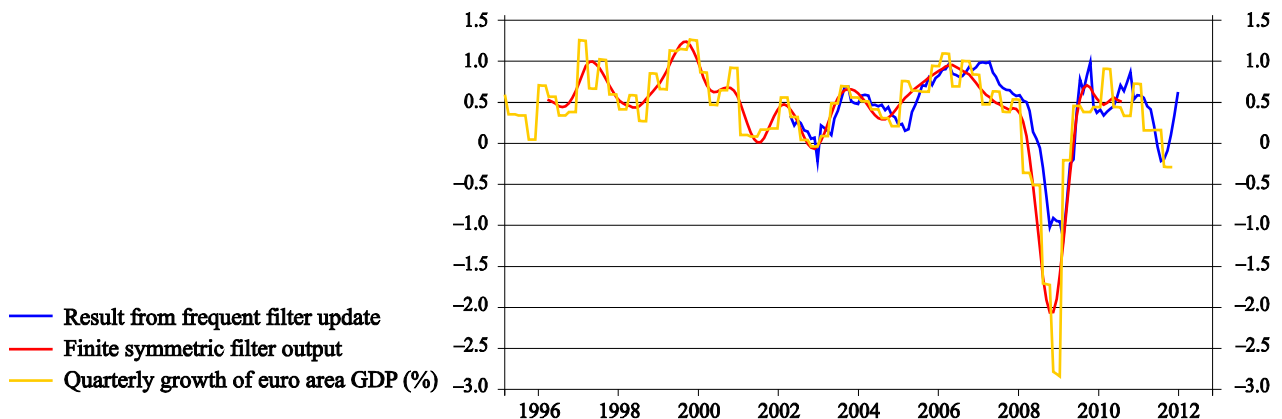
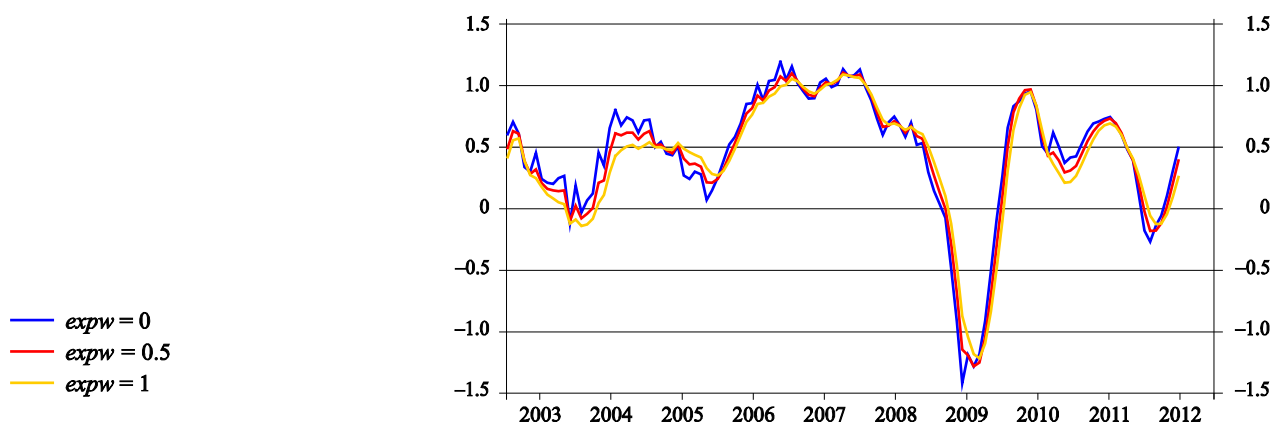


Figure 7

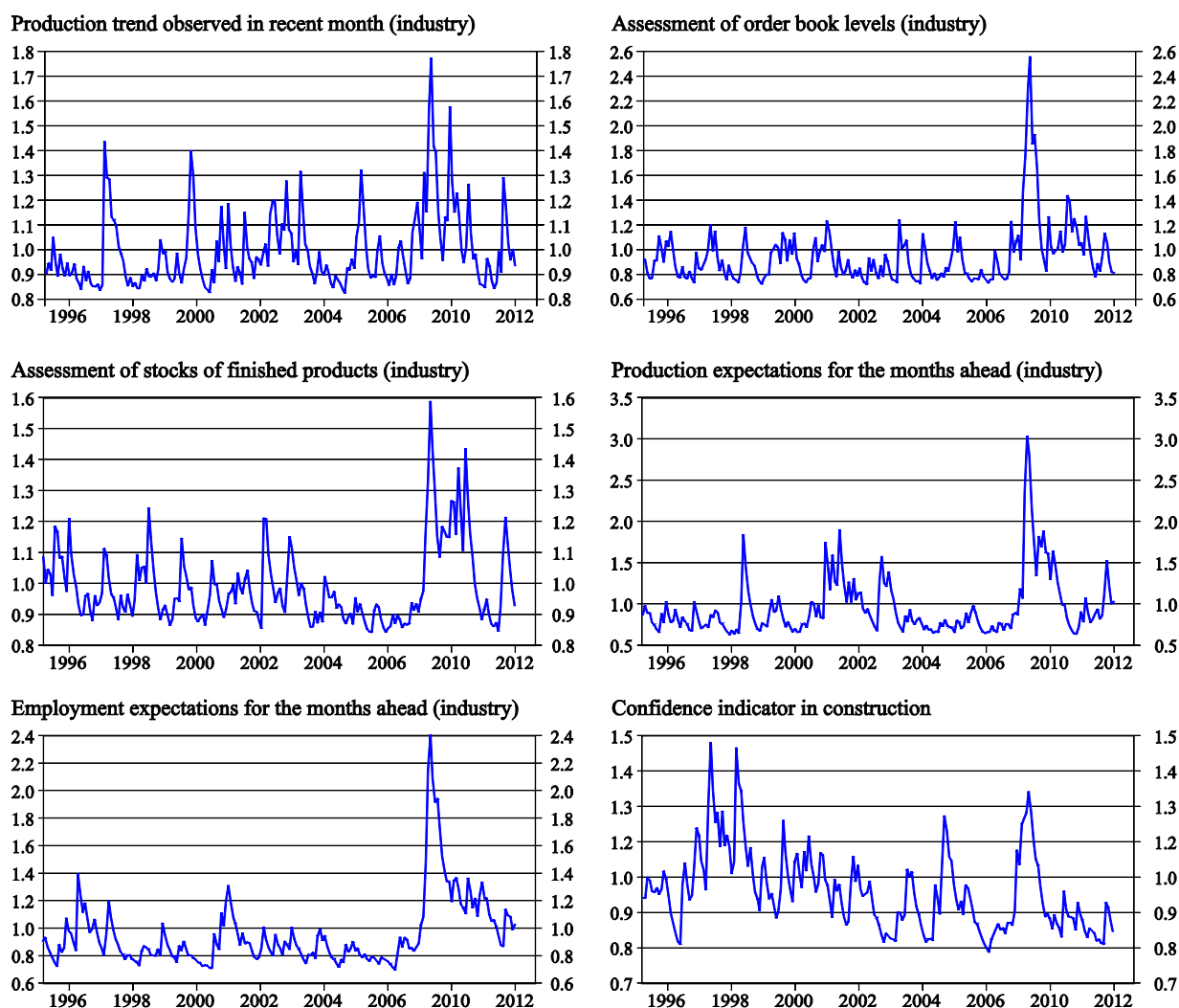
Three outputs from customised filter with different degrees of noise suppression in the rightmost stopband (%)



Notes: The blue line corresponds to the output of the classical mean squared error problem solution. The final indicator presented in this paper is the middle case with  $expw = 0.5$ .

Figure 8

GARCH(1,1) estimated standard deviations of GD Ecfm data



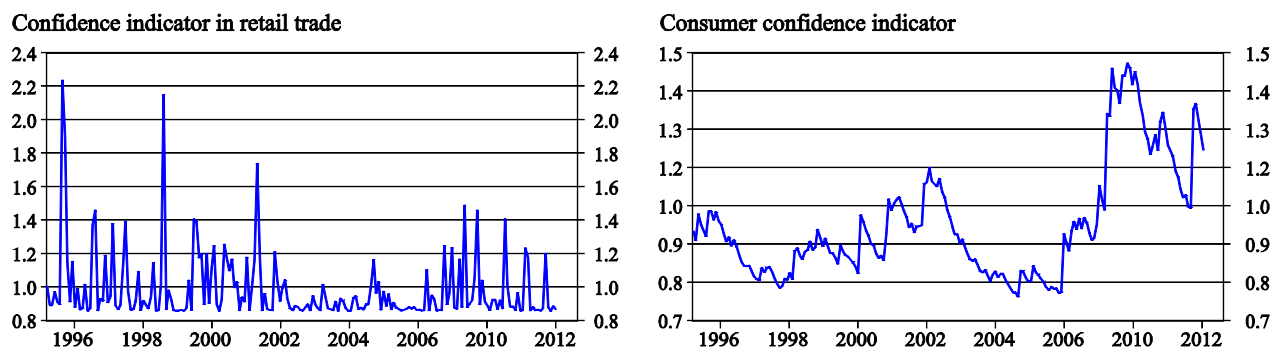


Figure 9  
Rounded **first** release vs **final** vintage of six Ecfm series

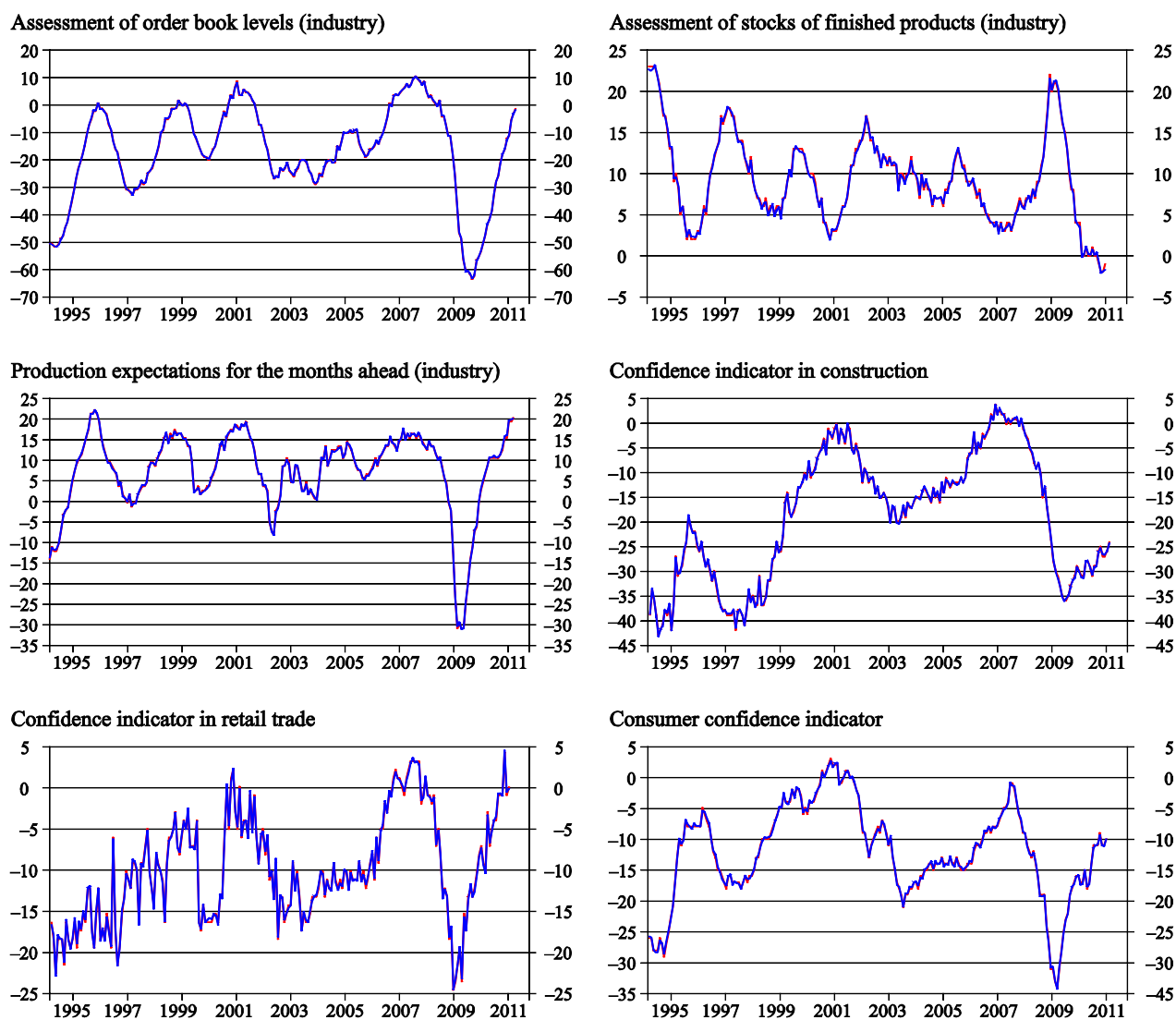




Figure 10

## Prototypical indicator for German GDP

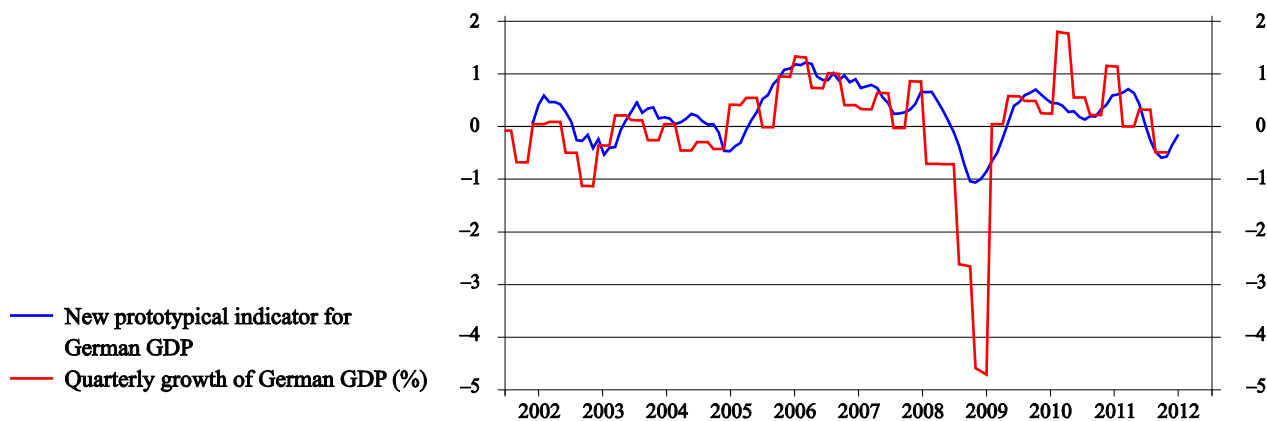


Figure 11

## Prototypical indicator for French GDP

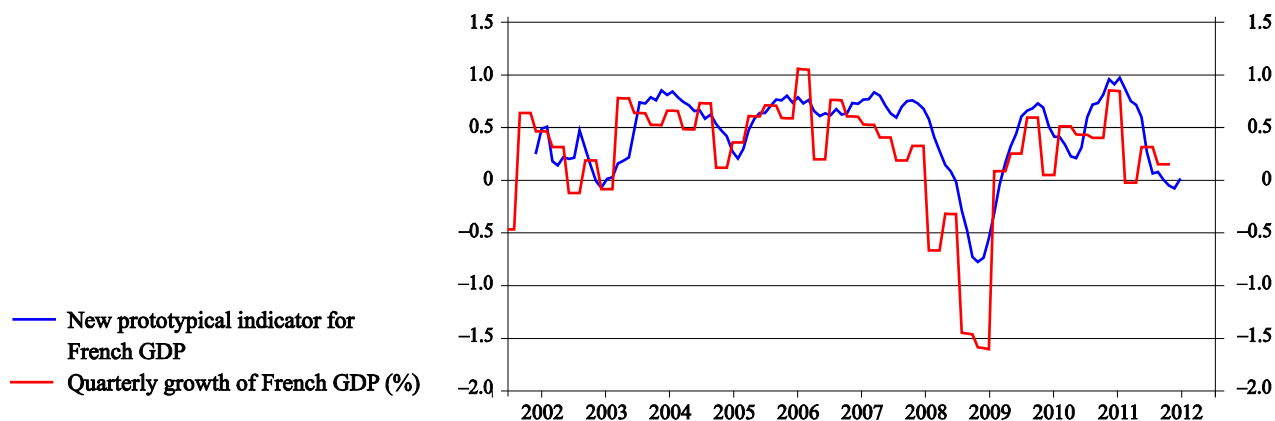
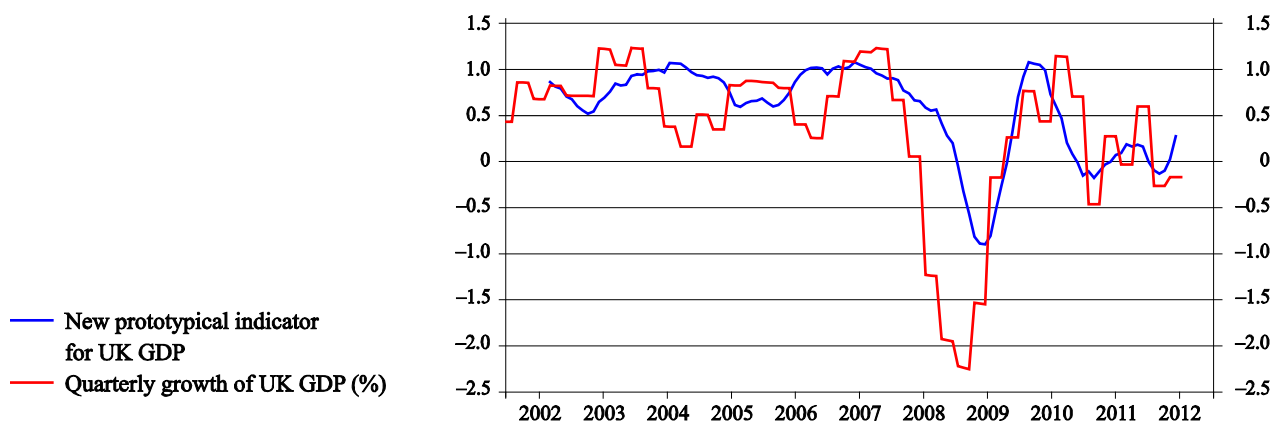


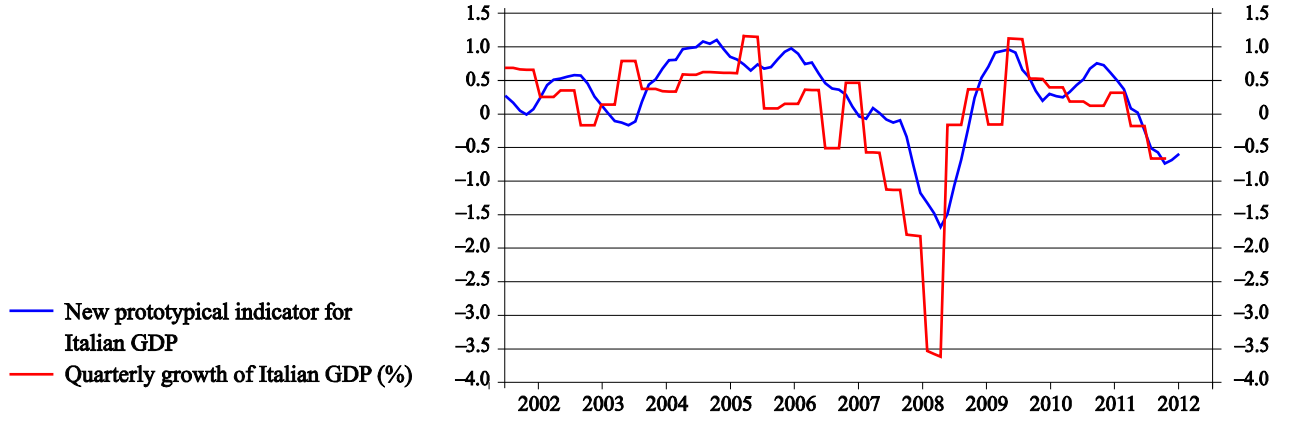
Figure 12

## Prototypical indicator for UK GDP



Note. The explanatory variables – UK business and consumer confidence data – are insufficient in explaining changes in UK GDP series.

Figure 13  
Prototypical indicator for Italian GDP



### Derivation of univariate direct filter approach

Expression (7) is rewritten as follows:

$$\begin{aligned} \sum_{k=-[T/2]}^{[T/2]} w_k \left| \Gamma(\omega_k) - \text{Re}(\hat{\Gamma}(\omega_k)) - i \text{Im}(\hat{\Gamma}(\omega_k)) \right|^2 I_{Tx}(\omega_k) W(\omega_k) = \\ = \sum_{k=-[T/2]}^{[T/2]} w_k \left( \left[ \Gamma(\omega_k) - \text{Re}(\hat{\Gamma}(\omega_k)) \right]^2 + \text{Im}(\hat{\Gamma}(\omega_k))^2 \right) I_{Tx}(\omega_k) W(\omega_k) \end{aligned} \quad (15).$$

Expression (15) is differentiated w.r.t. filter parameters:

$$\begin{aligned} \sum_{k=-[T/2]}^{[T/2]} w_k \left( (\Gamma(\omega_k) - \text{Re}(\hat{\Gamma}(\omega_k))) (-d/db_j (\text{Re}(\hat{\Gamma}(\omega_k)))) + \right. \\ \left. + \text{Im}(\hat{\Gamma}(\omega_k)) d/db_j (\text{Im}(\hat{\Gamma}(\omega_k))) \right) I_{Tx}(\omega_k) W(\omega_k) = 0 \end{aligned} \quad (16).$$

Since

$$\frac{d}{db_j} \text{Re}(\hat{\Gamma}(\omega_k)) = \cos(-j\omega_k) \quad (17)$$

and

$$\frac{d}{db_j} \text{Im}(\hat{\Gamma}(\omega_k)) = \sin(-j\omega_k) \quad (18)$$

expression (16) becomes

$$\sum_{k=-[T/2]}^{[T/2]} w_k \left( (\Gamma(\omega_k) - \text{Re}(\hat{\Gamma}(\omega_k))) (-\cos(-j\omega_k)) + \text{Im}(\hat{\Gamma}(\omega_k)) \sin(-j\omega_k) \right) I_{Tx}(\omega_k) W(\omega_k) = 0 \quad (19)$$

or

$$\begin{aligned}
& \sum_{k=-[T/2]}^{[T/2]} w_k \Gamma(\omega_k) \cos(-j\omega_k) I_{Tx}(\omega_k) W(\omega_k) = \\
& = \sum_{k=-[T/2]}^{[T/2]} w_k \left( \text{Re}(\hat{\Gamma}(\omega_k)) \cos(-j\omega_k) + \text{Im}(\hat{\Gamma}(\omega_k)) \sin(-j\omega_k) \right) I_{Tx}(\omega_k) W(\omega_k) \quad (20).
\end{aligned}$$

Since

$$\text{Re}(\hat{\Gamma}(\omega_k)) = \sum_l b_l \cos(-l\omega_k) \quad (21)$$

and

$$\text{Im}(\hat{\Gamma}(\omega_k)) = \sum_l b_l \sin(-l\omega_k) \quad (22)$$

the right-hand side of equation (20) is

$$\begin{aligned}
& b_0 \sum_{k=-[T/2]}^{[T/2]} w_k (\cos(-j\omega_k) \cos(-0\omega_k) + \sin(-j\omega_k) \sin(-0\omega_k)) I_{Tx}(\omega_k) W(\omega_k) + \\
& b_1 \sum_{k=-[T/2]}^{[T/2]} w_k (\cos(-j\omega_k) \cos(-1\omega_k) + \sin(-j\omega_k) \sin(-1\omega_k)) I_{Tx}(\omega_k) W(\omega_k) + \\
& \vdots \\
& b_L \sum_{k=-[T/2]}^{[T/2]} w_k (\cos(-j\omega_k) \cos(-L\omega_k) + \sin(-j\omega_k) \sin(-L\omega_k)) I_{Tx}(\omega_k) W(\omega_k) \quad (23).
\end{aligned}$$

Let  $b := [b_1, b_2, \dots, b_L]' = C^{-1}V$ .

Then, the right-hand side of equation (20) implies

$$C = \left( \sum_{k=-[T/2]}^{[T/2]} w_k (\cos(-j\omega_k) \cos(-l\omega_k) + \sin(-j\omega_k) \sin(-l\omega_k)) I_{Tx}(\omega_k) W(\omega_k) \right)_{jl} \quad (24)$$

where  $0 \leq j, l \leq L$ .

The left-hand side of equation (20) implies

$$V = \left( \sum_{k=-[T/2]}^{[T/2]} w_k \Gamma(\omega_k) \cos(-j\omega_k) I_{Tx}(\omega_k) W(\omega_k) \right)_j \quad (25)$$

where  $j = 0, \dots, L$ .

### Derivation of multivariate direct filter approach

The multivariate version of expression (14) is as follows:

$$\begin{aligned}
& \sum_{k=-[T/2]}^{[T/2]} w_k \left| \Gamma(\omega_k) \Xi_{Tx}(\omega_k) - Re \left( \hat{\Gamma}(\omega_k) \Xi_{Tx}(\omega_k) + \sum_{n=1}^m \hat{\Gamma}_{z_n}(\omega_k) \Xi_{Tz_n}(\omega_k) \right) - \right. \\
& \left. - iIm \left( \hat{\Gamma}(\omega_k) \Xi_{Tx}(\omega_k) + \sum_{n=1}^m \hat{\Gamma}_{z_n}(\omega_k) \Xi_{Tz_n}(\omega_k) \right) \right|^2 W(\omega_k) \\
& = \sum_{k=-[T/2]}^{[T/2]} w_k \left( \Gamma(\omega_k) Re(\Xi_{Tx}(\omega_k)) - Re(\hat{\Gamma}_x(\omega_k) \Xi_{Tx}(\omega_k)) - \sum_{n=1}^m Re(\hat{\Gamma}_{z_n}(\omega_k) \Xi_{Tz_n}(\omega_k)) \right)^2 W(\omega_k) + \\
& + \sum_{k=-[T/2]}^{[T/2]} w_k (\Gamma(\omega_k) Im(\Xi_{Tx}(\omega_k)) - [Im(\hat{\Gamma}_x(\omega_k) \Xi_{Tx}(\omega_k)) + \sum_{n=1}^m Im(\hat{\Gamma}_{z_n}(\omega_k) \Xi_{Tz_n}(\omega_k))])^2 W(\omega_k) \quad (26).
\end{aligned}$$

The argument of  $\Xi_{Tx}(\omega_k)$  is isolated outside the filter expression to get rid of nuisance term  $\Gamma(\omega_k) Im(\Xi_{Tx}(\omega_k))$  in the last line of expression (26):

$$\sum_{k=-[T/2]}^{[T/2]} w_k \left| \Gamma(\omega_k) \Xi_{Tx}(\omega_k) - Re(expr) - iIm(expr) \right|^2 \left| \exp(i \arg(\Xi_{Tx}(\omega_k))) \right|^2 W(\omega_k) \quad (27)$$

where

$$expr = \hat{\Gamma}_x(\omega_k) \Xi_{Tx}(\omega_k) + \sum_{n=1}^m \hat{\Gamma}_{z_n}(\omega_k) \Xi_{Tz_n}(\omega_k) \exp(-i \arg(\Xi_{Tx}(\omega_k))) \quad (28)$$

In expression (27), only  $\exp(i \arg(\Xi_{Tx}(\omega_k)))$  is isolated. Since  $|\exp(i \arg(\Xi_{Tx}(\omega_k)))|^2 = 1$ , expression (27) becomes

$$\sum_{k=-[T/2]}^{[T/2]} w_k \left| \Gamma(\omega_k) \Xi_{Tx}(\omega_k) - Re(expr) - iIm(expr) \right|^2 W(\omega_k) \quad (29).$$

In expression (29), the nuisance term has vanished in the imaginary part.

Denote

$$\begin{aligned}
\tilde{\Xi}_{Tx}(\omega_k) &= |\Xi_{Tx}(\omega_k)| \\
\tilde{\Xi}_{Tz_n}(\omega_k) &= \Xi_{Tz_n}(\omega_k) \exp(-i \arg(\Xi_{Tx}(\omega_k)))
\end{aligned} \quad (30)$$

Expression (29) is differentiated w.r.t.  $b_j^m$  (the j-th MA coefficient of the filter applied to  $z_{mt}$ ) and equated to zero:

$$\sum_{k=-[T/2]}^{[T/2]} w_k \left( \Gamma(\omega_k) \tilde{\Xi}_{Tx}(\omega_k) - Re(\hat{\Gamma}_x(\omega_k) \tilde{\Xi}_{Tx}(\omega_k)) - \sum_{n=1}^m Re(\hat{\Gamma}_{z_n}(\omega_k) \tilde{\Xi}_{Tz_n}(\omega_k)) \right) \times$$

$$\begin{aligned}
& \times (-1) \operatorname{Re} \left( \exp(ij\omega_k) \tilde{\Xi}_{Tz_n}(\omega_k) \right) W(\omega_k) \\
& - \sum_{k=-[T/2]}^{[T/2]} w_k \left( \left[ \operatorname{Im} \left( \hat{\Gamma}_x(\omega_k) \tilde{\Xi}_{Tx}(\omega_k) \right) + \sum_{n=1}^m \operatorname{Im} \left( \hat{\Gamma}_{z_n}(\omega_k) \tilde{\Xi}_{Tz_n}(\omega_k) \right) \right] \right) \times \\
& \times (-1) \operatorname{Im} \left( \exp(ij\omega_k) \tilde{\Xi}_{Tz_n}(\omega_k) \right) W(\omega_k) = 0
\end{aligned} \tag{31}$$

where  $\tilde{\Xi}_{Tz_m}(\omega_k) = \tilde{\Xi}_{Tx}$  if  $m = 0$ .

Rearranging left-hand side equation (31), we obtain

$$\begin{aligned}
& \sum_{k=-[T/2]}^{[T/2]} w_k \Gamma(\omega_k) \tilde{\Xi}_{Tx}(\omega_k) \operatorname{Re} \left( \exp(ij\omega_k) \tilde{\Xi}_{Tz_m}(\omega_k) \right) W(\omega_k) = \\
& = \sum_{k=-[T/2]}^{[T/2]} w_k \left( \operatorname{Re} \left( \hat{\Gamma}_x(\omega_k) \tilde{\Xi}_{Tx}(\omega_k) \right) + \sum_{n=1}^m \operatorname{Re} \left( \hat{\Gamma}_{z_n}(\omega_k) \tilde{\Xi}_{Tz_n}(\omega_k) \right) \right) \operatorname{Re} \left( \exp(ij\omega_k) \tilde{\Xi}_{Tz_m}(\omega_k) \right) W(\omega_k) + \\
& + \sum_{k=-[T/2]}^{[T/2]} w_k \left( \operatorname{Im} \left( \hat{\Gamma}_x(\omega_k) \tilde{\Xi}_{Tx}(\omega_k) \right) + \sum_{n=1}^m \operatorname{Im} \left( \hat{\Gamma}_{z_n}(\omega_k) \tilde{\Xi}_{Tz_n}(\omega_k) \right) \right) \operatorname{Im} \left( \exp(ij\omega_k) \tilde{\Xi}_{Tz_m}(\omega_k) \right) W(\omega_k) \tag{32}.
\end{aligned}$$

Equation (32) reduces to univariate mean squared criterion when  $m = 0$ ; the term in l.h.s. of equation (32) becomes

$$w_k \Gamma(\omega_k) \tilde{\Xi}_{Tx}(\omega_k) \operatorname{Re} \left( \exp(ij\omega_k) \tilde{\Xi}_{Tz_m}(\omega_k) \right) W(\omega_k) = w_k \Gamma(\omega_k) \cos(-j\omega_k) I_{Tx}(\omega_k) W(\omega_k) \tag{33},$$

which corresponds to expression (25). The r.h.s. simplifies to

$$\begin{aligned}
& w_k \operatorname{Re} \left( \hat{\Gamma}_x(\omega_k) \tilde{\Xi}_{Tx}(\omega_k) \right) \operatorname{Re} \left( \exp(ij\omega_k) \tilde{\Xi}_{Tx}(\omega_k) \right) W(\omega_k) + \\
& + w_k \operatorname{Im} \left( \hat{\Gamma}_x(\omega_k) \tilde{\Xi}_{Tx}(\omega_k) \right) \operatorname{Im} \left( \exp(ij\omega_k) \tilde{\Xi}_{Tx}(\omega_k) \right) W(\omega_k) = \\
& = w_k \operatorname{Re} \left( \hat{\Gamma}_x(\omega_k) \tilde{\Xi}_{Tx}(\omega_k) \right) \operatorname{Re} \left( \overline{\exp(ij\omega_k) \tilde{\Xi}_{Tx}(\omega_k)} \right) W(\omega_k) - \\
& - w_k \operatorname{Im} \left( \hat{\Gamma}_x(\omega_k) \tilde{\Xi}_{Tx}(\omega_k) \right) \operatorname{Im} \left( \overline{\exp(ij\omega_k) \tilde{\Xi}_{Tx}(\omega_k)} \right) W(\omega_k) = \\
& = w_k \operatorname{Re} \left( \hat{\Gamma}_x(\omega_k) \overline{\exp(ij\omega_k)} \right) I_{Tx}(\omega_k) W(\omega_k)
\end{aligned} \tag{34},$$

which corresponds to expression (24), and where an overbar denotes a complex conjugate. The r.h.s. of equation (32) attributes the following weight to the filter coefficient  $b_l''$ :

$$\begin{aligned}
& w \operatorname{Re} \left( \exp(il\omega_k) \Xi_{Tz_u}(\omega_k) \right) \operatorname{Re} \left( \exp(ij\omega_k) \Xi_{Tz_m}(\omega_k) \right) W(\omega_k) + \\
& + w_k \operatorname{Im} \left( \exp(il\omega_k) \Xi_{Tz_u}(\omega_k) \right) \operatorname{Im} \left( \exp(ij\omega_k) \Xi_{Tz_m}(\omega_k) \right) W(\omega_k)
\end{aligned} \tag{35}$$

where  $\Xi_{Tz_u}(\omega_k) = \Xi_{Tx}(\omega_k)$  for  $u = 0$ .

This generalised  $C$  matrix reduces to expression (24) in univariate case.

### First order constraint

The filter is subject to the first order constraint

$$b_1^n + b_2^n + \dots + b_L^n = w^n \quad (36)$$

imposes amplitude constraint in frequency zero according to  $\hat{\Gamma}_{z_n}(0) = w^n$  ( $\hat{\Gamma}_{z_n} = \hat{\Gamma}_x$  if  $m = 0$ ), and  $w^n$  is set to unity.

### GARCH normalisation

Variance at period  $t$ ,  $h_t$ , of a demeaned input variable  $x_t$  is modeled with a GARCH(1,1) process:

$$h_t = \alpha_0 + \alpha_1 x_{t-1}^2 + \beta_1 h_{t-1} \quad (37)$$

where  $\alpha_0 > 0$ ,  $\alpha_1 \geq 0$ ,  $\beta_1 \geq 0$  and assumed  $x_t : N(0, h_t)$ . The Kolmogorov-Smirnov (KS) test for normality (see, e.g. Massey (1951)) fails to reject the null hypothesis of (standard) normal distribution for all input variables at 5% significance. However, it is well known that KS test has low power for small sample sizes. An alternative, the Jarque-Bera test (Jarque and Bera (1987)) rejects the null of normality for 5 of 8 input series, at 5% significance. Therefore, the assumption  $x_t : N(0, h_t)$  appears not to hold. Nonetheless, GARCH(1,1) estimated by quasi-maximum likelihood is found to be generally consistent under mild assumptions, see Bollerslev and Wooldridge (1992).

Table 1

#### The list of data used for the construction of indicators

Variable	Source	Euro area	Germany	France	UK	Italy	Transformation
Real GDP, SA	Eurostat	x	x	x	x	x	$\Delta \log$ , lin. interp.
Production trend observed in recent month (industry), SA	DG Ecfin	x	x	x	x	x	$\Delta$
Assessment of order book levels (industry), SA	DG Ecfin	x	x	x	x	x	$\Delta$
Assessment of stocks of finished products (industry), SA	DG Ecfin	x	x	x	x	x	$\Delta$
Employment expectations for the months ahead (industry), SA	DG Ecfin	x	x	x	x	x	$\Delta$
Production expectations for the months ahead (industry), SA	DG Ecfin	x	x	x	x	x	$\Delta$
Confidence indicator in construction, SA	DG Ecfin	x	x	x	x	x	$\Delta$
Confidence indicator in retail trade, SA	DG Ecfin	x	x	x	x	x	$\Delta$
Consumers confidence indicator, SA	DG Ecfin	x	x	x	x	x	$\Delta$
Confidence indicator in services, SA	DG Ecfin		x				$\Delta$
Euro area share price index	Eurostat		x		x	x	$\Delta \log$
The US share price index	Eurostat	x	x			x	$\Delta \log$

Note: Confidence indicators and GDP data correspond to particular regions.



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