Optimal fiscal policy in a DSGE model with heterogeneous agents[*]

Jonathan Swarbrick[†]

School of Economics, University of Surrey

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Abstract

This paper solves optimal fiscal policy in a simple real business cycle model with agent heterogeneity. Introducing *rule-of-thumb* behaviour in some agents, I show that the steady state optimal tax on capital in the long run should always be zero regardless of the governments favouritism towards particular agents. Over the business cycle, the inclusion of *rule-of-thumb* behaviour had a significant effect on optimal stabilisation policy. The tax rate changes implemented to stabilise the economy after shocks were contrary to those recommended by a representative agent version of the model . Following a productivity shock, the government should finance a short term subsidy on capital with additional labour taxes. This is at odds with the representative agent model that recommended a labour tax reduction financed by a short term tax on capital. It was also found that temporary increases to government expenditure should be financed by short term taxes on capital. Again this was contrary to the representative agent model which recommended that additional spending should be financed with labour tax increases. Whilst forming an insightful analysis, the study is not sufficient for making policy recommendations. Endogenising the agent heterogeneity and introducing imperfections such as nominal rigidities would go some way to developing a suitable policy tool.

JEL-Classification: E32, E62, H20, H31
Keywords: Rule of thumb, optimal fiscal policy, welfare.

1 Introduction

This paper addresses the following question: how should taxes be set to maximise welfare when considering inequality? This then provokes another challenging question; what is the appropriate measure of welfare in the presence of inequality in society?

To answer these questions I build a calibrated two-agent real business cycle model and optimise for different weightings in a utilitarian measure of welfare. This allows me to suggest optimising fiscal policy for a government favouring the poor, policy for a government favouring the rich and one for an average measure of welfare both in the long run and over the business cycle.

Over the last 12 months, many advanced economies have experienced social unrest and protest movements motivated primarily by inequality in society. With growing debate surrounding the role of fiscal policy in distributive issues it has been speculated that representative agent models are not sufficient to analyse the welfare effects of fiscal policy (e.g. Domeij & Heathcote 2004). One of the significant issues relates to the composition of taxation. Representative agent models typically recommend a zero tax on capital and whilst I confirm that introducing a second agent into a simple RBC model does not change this proposition, the presence of agent heterogeneity allows for a richer analysis of how fiscal policy should be conducted over the business cycle and also of the distributional considerations of tax reform.

I incorporate a second type of agent to the model with the introduction of *rule-of-thumb* behaviour. This allows the analysis of the effects of changes to fiscal policy in a more realistic context of inequality between the 'savers' and the 'spenders'. Modelling consumers that do not borrow or save follows empirical evidence in the literature that a significant proportion of people live month to month, many with virtually no wealth (Mankiw 2000). In doing so I find that the optimal tax on capital should be zero in the long run regardless of the government preference towards the agent types. To finance an exogenous level of spending, the government impose a tax on labour. Testing the impact of both demand and supply shocks, I find that the presence of the second type of agent does significantly affect how fiscal policy should be conducted over the business cycle compared to a benchmark representative model.

The paper contributes to an interesting conversation. The presence of *rule-of-thumb* agents do not affect the composition of long-run optimal tax rates but do affect how

taxes should be set over the business cycle. I find that the paper could be further enriched by relaxing some of the assumptions such as a balanced government budget, and introducing frictions into the model. And so looking forward, the paper ends with a discussion of potential areas for extending the study.

2   Background Discussion

Kumhof & Yakadina (2010) argue that whilst optimal monetary policy has developed to the point of being able to make highly relevant policy recommendations, the same cannot be said of fiscal policy. There are persistent disagreements around questions of efficacy (Auerbach 2002); whether the multiplier exists and how big it is (Ilzetzki, Mendoza & Végh 2010); the validity of Ricardian equivalence (Stanley 1998); optimal debt levels (Kumhof & Yakadina 2010); the composition of optimal taxation (Domeij & Heathcote 2004) and so on.

Whilst there is a substantial body of literature examining the role of fiscal policy, the analysis has been dominated by the use of representative agent models. Krusell & Smith's (1998) seminal paper on heterogeneity showed that whilst aggregating consumer behaviour was sufficient for the analysis of most macroeconomic variables, in order to study distributional effects of policy it was necessary to model different types of agent. Despite this, the computational demands for simplicity have led to the widespread use of representative agents in optimising DSGE models. The implications of this on government policy have been significant with some (e.g. Smith 2012) suggesting that central banks and governments have often adopted the simplest models rather than the most realistic. Recent research using heterogeneous agents has exposed the limitations of using these aggregate models for policy analysis. Domeij & Heathcote (2004) for instance, show that whilst cutting capital income tax increases the welfare of a representative economy, in the heterogeneous agent economy only the very rich benefit and most households would experience large welfare losses.

Considering appropriate methods of modeling a heterogeneous economy, Mankiw (2000) highlights three observations from empirical research. Firstly, Mankiw argues that consumption smoothing is far from perfect in that a significant proportion of the population spend the majority of their income whilst saving very little. His second observation is that approximately one fifth of households have zero or negative wealth. Exclud-

ing home equity, this figure rises to nearly 30%. Finally, whilst there are those with very little wealth, Mankiw argues that there is a proportion of society whose wealth is very large, far beyond that for consumption smoothing but rather is used as bequest. These three points lead Mankiw to suggest that new models are needed that acknowledge these two main groups; those that have very little wealth and spend all their income every month - the spenders - and those that hold much wealth - the savers. In a model featuring these 'spenders' or *rule-of-thumb* consumers, Galí, López-Salido & Vallés (2007) are able to more accurately represent consumption behaviour in response to changes to government spending.

Ábrahám & Cárceles-Poveda (2010) suggest that in order to study the welfare effect of fiscal reform, it is necessary to model a realistic wealth distribution. For this reason, this study models the behaviour of an economy featuring both Ricardian 'wealthy' agents and *rule-of-thumb* 'poor' agents.

There are, however, contrasting optimal policy recommendations using heterogeneous agent models. Chamley (1986) demonstrates that the optimal tax on capital in the long run should be zero even in the presence of heterogeneous agents. Whilst levying tax on capital is often considered an appropriate instrument for redistribution, the author argues that doing so is not an efficient policy in the long run. Building a model featuring both *rule-of-thumb* and Ricardian consumers, Colciago (2007) observed that a fiscal reform including a reduction in capital taxes would both improve the long-run economic activity and would benefit the non capital holding agents if accompanied by effective redistributive policies. The position of these studies has however been challenged by others. Aiyagari (1995) for instance, finds that with incomplete insurance markets and borrowing constraints, the optimal capital tax rate is positive, even in the long run.

Of course whilst fiscal policy in the unrealistic steady state is a useful conversation, another important question with significant welfare consequences is how fiscal policy should be conducted over the business cycle. Attempting to answer this questions using a neoclassical model, Chari & Christiano (1993) find that labour taxes fluctuate very little whilst taxes on private assets fluctuate a great deal. Blanchard (2009) suggests that the welfare costs of fluctuations tend to fall on different agents within the economy disproportionately and so this paper extends the core of Chari & Christiano's (1993) analysis to a model with inequality to find how tax rates are optimised over the business cycle.

3   A Representative Benchmark

3.1   The Model

I consider a benchmark economy that comprises a representative household, a firm and a government. The infinitely lived household receives income from providing labour and capital and chooses a path of consumption and capital investment to maximise their utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t) \tag{3.1}$$

where C_t and H_t are household consumption and labour provided at time t. It is assumed that utility is in the logarithmic form

$$U_t = U(C_t, H_t) = \ln(C_t) + \chi \ln(1 - H_t). \tag{3.2}$$

The government taxes both labour and capital income at τ_H and τ_K respectively. The intertemporal budget then is described by equation 3.3 where K_t is the capital stock, W_t is the wage rate and R_t the capital rental rate as described below, δ is the capital depreciation rate. The labour supply and Euler equations are shown in 3.4 and 3.5

$$C_t + K_{t+1} = (1 - \tau_H)W_t H_t + (1 + (1 - \tau_K)R_t - \delta)K_t \tag{3.3}$$

$$(1 - \tau_H)W_t = \chi \frac{C_t}{(1 - H_t)} \tag{3.4}$$

$$\frac{1}{C_t} = \beta E \left\{ (1 + (1 - \tau_K)R_t - \delta)\frac{1}{C_{t+1}} \right\} \tag{3.5}$$

A representative firm rents capital and employs labour to produce a single consumption good. I assume a Cobb-Douglas production function and zero profits so that capital and labour are paid at their marginal products.

$$Y_t = F(H_t, K_t) = (e^{a_t} H_t)^{\alpha} K_t^{1-\alpha} \tag{3.6}$$

$$W_t = \alpha(e^{a_t})^{\alpha} \left(\frac{H_t}{K_t} \right)^{\alpha - 1} \tag{3.7}$$

$$R_t = (1 - \alpha)(e^{a_t})^{\alpha} \left(\frac{H_t}{K_t} \right)^{\alpha} \tag{3.8}$$

e^{a_t} represents the labour augmented technical progress with a following an AR(1) process to allow the analysis of the effects of productivity or supply shocks.

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \tag{3.9}$$

The capital stock evolves according to the law of motion

$$K_{t+1} = I_t + (1-\delta)K_t. \tag{3.10}$$

The government imposes taxes on household income from labour and capital to finance exogenously given consumption. It is assumed that the government must operate with a balanced budget. The budget constraint can then be expressed as

$$e^{g_t}G_t = \tau_H W_t H_t + \tau_K R_t K_t \tag{3.11}$$

Government spending G_t is exogenously given and g_t is used to represent exogenous shocks to government spending. Similarly to a_t, g_t evolves according to the AR(1) process

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}. \tag{3.12}$$

3.2 Fiscal Policy

The government solves the Ramsey problem by optimizing the social welfare function accounting for firm and household behaviour, setting fiscal policy under commitment. Assuming the the government objective function is equal to the representative household lifetime utility, a competitive equilibrium of government policy, allocations and prices is defined such that the following conditions hold.

1. Households maximise their utility (3.1) subject to 3.3 such that 3.4 and 3.5 hold.

2. Firms maximise profits such that 3.7 and 3.8 hold.

3. The government budget is balanced (3.11).

With a representative agent and no frictions the optimal fiscal policy will set distortionary taxes to zero. With exogenous government spending the revenue can be shown to be raised solely from labour income taxes keeping capital income tax at zero. In a similar manner to Ljungqvist & Sargent (2004) and following the above conditions, the

Lagrangian of the Ramsey problem can be expressed[1]

$$
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{
\begin{array}{c}
U_t \\
+ \underbrace{\lambda_t \left(F\left(K_t, H_t\right) - C_t - e^{g_t} G_t - K_{(t+1)} + (1-\delta) K_t \right)}_{\text{Resource Constraint}} \\
+ \underbrace{\varphi_t \left(F\left(K_t, H_t\right) - (1 - \tau_{K,t}) R_t K_t - (1 - \tau_{H,t}) W_t H_t - e^{g_t} G_t \right)}_{\text{Government Budget Constraint}} \\
+ \underbrace{\phi_t \left(U_{H,t} - U_{C,t} (1 - \tau_{H,t}) W_t \right)}_{\text{Labour Supply}} \\
+ \underbrace{\psi_t \left(U_{C,t} - \beta U_{C,t+1} ((1 - \tau_{K,t}) R_t + 1 - \delta) \right)}_{\text{Euler Equation}}
\end{array}
\right\}
$$

(3.13)

Taking the first order condition with respect to K_{t+1} leads to

$$
\lambda_t = \beta[\lambda_{t+1}(F_{K,t+1} + (1-\delta)) + \varphi_{t+1}(F_{K,t+1} - (1 - \tau_{K,t+1})R_{t+1})]
$$

(3.14)

which in steady state could be written

$$
\lambda = \beta[\lambda(R + 1 - \delta) + \varphi(R - (1 - \tau_K)R)].
$$

(3.15)

Taking the Euler equation in steady state and subbing into this condition leads to

$$
\lambda = \lambda\beta(R + 1 - \delta) + \varphi\beta(R - (1 - \tau_K)R).
$$

(3.16)

which then simplifies to

$$
(\varphi + \lambda)(R - (1 - \tau_K)R) = 0.
$$

(3.17)

As in Ljungqvist & Sargent (2004), λ is interpreted as the marginal social value of goods and is strictly positive. φ is the marginal value of reducing taxes and is nonnegative. This implies that $\tau_K = 0$ in steady state. In keeping with previous literature, in a representative agent economy the optimal tax on capital should equal zero in steady state. In this case, government spending should therefore be financed solely from tax on labour income.

This finding was confirmed running the model in dynare as discussed below. This finding of course is only relevant to the economy in steady state and says nothing about the composition of taxes over the business cycle.

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[1]The derivation of the consumer first order conditions and the constraints used to construct the Lagrangian can be found in appendix **??**. The first order conditions from the Lagrangian can be found in appendix **??**.

3.3 Model Calibration

The model was calibrated to match UK data; the labour share, discount factor and depreciation rate were taken from Faccini, Millard & Zanetti (2011) and shock persistence and standard deviation are as were estimated in Harrison & Oomen (2010, p.42). Government spending was crudely calibrated to be around 40% of total output (HM Treasury. 2012, p.65). These parameter values are shown in table 1.

Table 1: Model parameter calibrations

Parameter	Value	Description
β	0.99	Discount Factor
α	0.69	Labour Share
δ	0.025	Depreciation Rate
ρ_a	0.89	Technology Shock Persistence
σ_a	0.006	Technology Shock Standard Deviation
ρ_g	0.96	Government Spending Shock Persistence
σ_g	0.008	Government Spending Shock Standard Deviation
G	0.5	Government Spending

3.4 Analysis

The dynare code of the model is shown in appendix **??**. As expected, the model optimized the tax rate on capital at a negative value very close to zero. As a consequence the government spending was financed entirely by levying a tax on labour.

3.4.1 Demand Shock

The economy displays the usual features following a shock to government spending such as a crowding out effect on consumption and investment. The decreased consumption leads to households choosing work over leisure as the marginal utility of consumption increases. The additional supply of labour causes a fall in wage. The drop in investment leads to a fall in capital stock and rise in rental rate.

With the assumption of a balanced budget, the spending is financed through increasing taxes. As shown in figure 1, it is optimal for taxes on capital income to fall, or rather

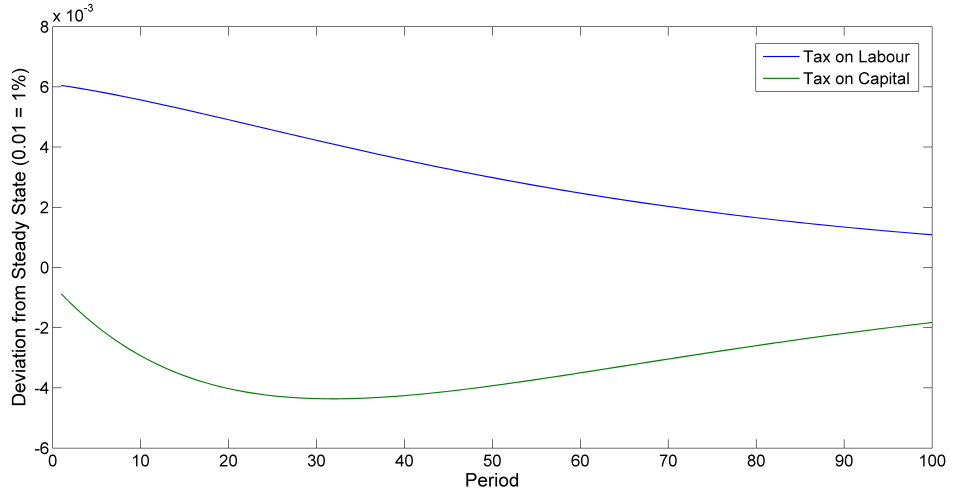to receive a small subsidy and for labour taxes to be increased. Studying the change in

Figure 1: Tax rates following government spending shock e^{g_t}

capital stock and labour supply provides insight into the movements in tax rates. The response of these variables together with rental rate and wage is plotted in figure 2. The
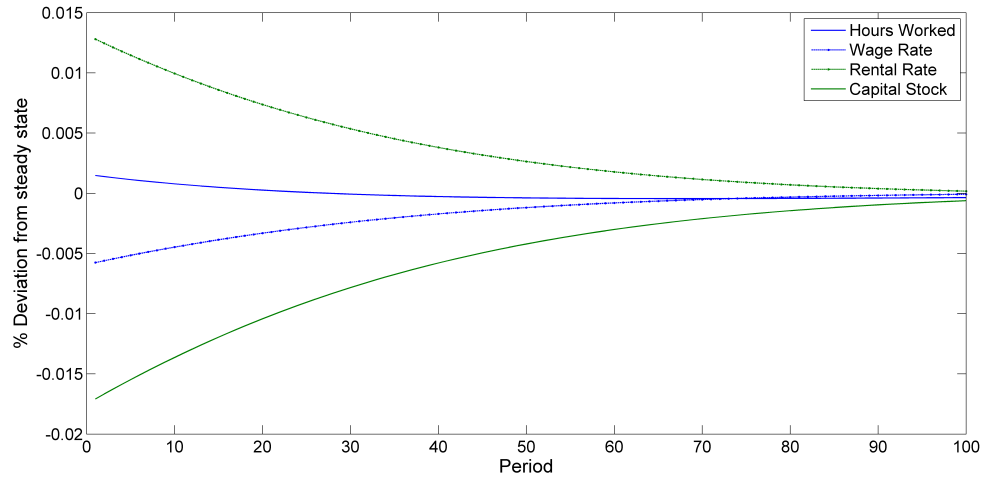

Figure 2: Wage, hours worked, rental rate and capital stock following government spending shock e^{g_t}

capital stock falls following a fall in investment; the small capital subsidy and increased rental rate provides incentive to return to the steady state value. As households are willing to provide more labour following the demand shock, it is beneficial for the government to finance the extra spending by increasing taxes on labour.

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3.4.2 Supply Shock

As would be expected, a positive shock to productivity leads to an increase of output for given factors of production. As well as output rising, the shock causes consumption and investment both to increase. With an increase in capital stock, rental rate falls below the steady state rate. Output increases more than the labour supply and so the wage rate increases. The households respond to higher wages by supplying more hours.
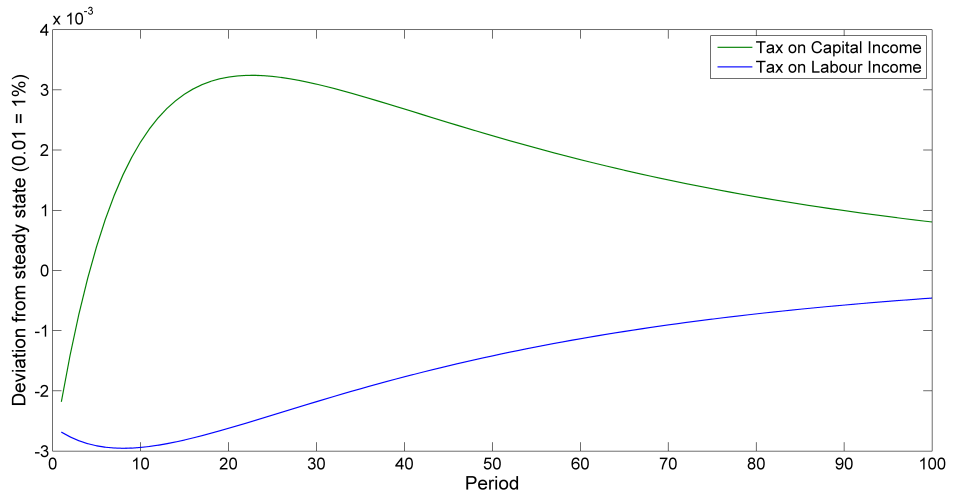

Figure 3: Tax rates following productivity shock e^{a_t}

The optimal response of the tax rates are plotted in figure 3. Government spending is unchanged following the shock and so taxes must adjust to respond to changes in hours worked and capital stock and the wage and rental rates. These are plotted on figure 4. In this case, both taxes fall initially but the increase in capital stock above the steady state value provides an incentive to tax return on capital. The labour supply also increases although only by a small amount. As the government tax revenue must remain unchanged, the tax on labour is actually reduced to offset the additional revenue from capital tax.

Similarly to the demand shock, it is optimal for the government to adjust both tax rates following the productivity shock. Under the assumption of a balanced budget, the government is unable to go into surplus and so must reduce revenue. The optimal tax on capital increases with the rise in capital stock. This might be even more pronounced if the initial periods following the shocks can be reinterpreted in the presence of nominal rigidities. In many New Keynesian models, wage is slow to increase
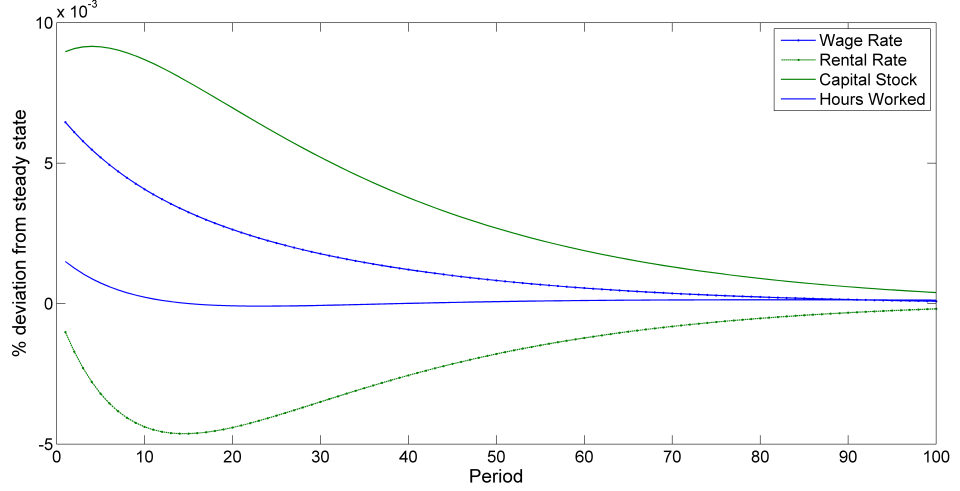
Figure 4: Wage, rental rate, capital stock and hours worked following productivity shock e^{a_t}

and hours worked can indeed fall. This is likely to change the initial optimal response significantly, perhaps with a greater increase in capital tax and a greater reduction in labour tax.

The analysis of both these two shocks give some insight into how fiscal policy should be conducted over the business cycle. The absence of nominal rigidities and the constraint of a balanced government budget create significant limitations. Nominal rigidities could lead to variables responding differently following the shocks and the presence of government debt might cause taxes to be conducted differently over the business cycle.

4   A Model with Inequality

Following others such as Mankiw (2000) and Galí et al. (2007) I introduce a second type of agent in the form of *rule-of-thumb* consumers. This type of agent holds no wealth and consumes all their income each period. These will be referred to as the 'poor' agents. The other 'wealthy' agents own all the capital and are able to smooth consumption as standard Ricardian agents. The poor will make up a proportion $\theta < 1$ of the population whilst the wealthy will make up the remaining $(1 - \theta)$ of the population. A superscript W denotes variables relating to the wealthy agent and P to the poor.

4.1 The Model

The economy comprises of two types of infinitely lived households, a firm and a government. The wealthy households receive income from providing labour and capital to the firm and choose a path of consumption and capital investment to maximise their expected lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U^W(C_t^W, H_t^W) \tag{4.1}$$

$$U_t^W = \ln(C_t^W) + \chi \ln(1 - H_t^W) \tag{4.2}$$

subject to the budget constraint

$$C_t^W + K_{t+1}^W = (1 - \tau_H)W_t H_t^W + (1 + R_t(1 - \tau_K) - \delta)K_t^W \tag{4.3}$$

where C_t^W and H_t^W are household consumption and labour provided at time t. K_t is the capital stock, W_t is the wage rate, R_t the rental rate and δ the capital depreciation rate. The government taxes both labour and capital income at τ_H and τ_K respectively. The capital stock evolves according to the law of motion

$$K_{t+1} = I_t + (1 - \delta)K_t. \tag{4.4}$$

Solving the first order conditions[2] gives the labour supply (4.5) and the Euler equation (4.6)

$$(1 - \tau_h)W_t = \chi \frac{C_t^W}{(1 - H_t^W)} \tag{4.5}$$

$$\frac{1}{C_t^W} = \beta E \left\{ (1 + (1 - \tau_K)R_t - \delta)\frac{1}{C_{t+1}^W} \right\} \tag{4.6}$$

The poor households receive income solely from labour and choose the number of hours worked each period to maximise utility given by

$$U_t^P = \ln(C_t^P) + \chi \ln(1 - H_t^P) \tag{4.7}$$

subject to the budget constraint

$$C_t^P = (1 - \tau_H)W_t H_t^P + T_t^P \tag{4.8}$$

where T_t^P is tax financed transfers from the government. This leads to a poor agent labour supply condition

$$(1 - \tau_h)W_t = \chi \frac{C_t^P}{(1 - H_t^P)}. \tag{4.9}$$

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[2]Full derivation for this model is outlined in appendix B

A representative firm rents capital and employs labour to produce consumption goods. I again assume a Cobb-Douglas production function and zero profits so that capital and labour are paid at their marginal products.

$$Y_t = F(H_t, K_t) = (e^{a_t} H_t)^\alpha K_t^{1-\alpha} \tag{4.10}$$

$$W_t = \alpha(e^{a_t})^\alpha \left(\frac{H_t}{K_t}\right)^{\alpha-1} \tag{4.11}$$

$$R_t = (1-\alpha)(e^{a_t})^\alpha \left(\frac{H_t}{K_t}\right)^\alpha \tag{4.12}$$

The variables K_t, H_t and C_t are aggregate variables given by

$$C_t = \theta C_t^P + (1-\theta)C_t^W \tag{4.13}$$

$$H_t = \theta H_t^P + (1-\theta)H_t^W \tag{4.14}$$

$$K_t = (1-\theta)K_t^W \tag{4.15}$$

Similar to the representative agent model, e^{a_t} represents the labour augmented technical progress with a following an AR(1) process to allow the analysis of the effects of productivity or supply shocks.

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \tag{4.16}$$

The government imposes taxes on household income from labour and capital to finance transfers to the poor agents and an exogenously given level of consumption. It is assumed that the government must operate with a balanced budget. The budget constraint can then be expressed as

$$e^{g_t} G_t + T_t = \tau_H W_t H_t + \tau_K R_t K_t \tag{4.17}$$

where $T_t = \theta T_t^P$. Again as in the previous model, g_t is used to represent exogenous shocks to government spending and evolves according to

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}. \tag{4.18}$$

Every period, the economy is bound by the resource constraint

$$Y_t = C_t + I_t + e^{g_t} G_t. \tag{4.19}$$

The equilibrium conditions which are used to solve the model can then be stated as:

$$\text{Poor Labour Supply:} \quad (1 - \tau_h)W_t = \chi \frac{C_t^P}{(1 - H_t^P)} \tag{4.20a}$$

$$\text{Poor Budget Constraint:} \quad C_t^P = (1 - \tau_H)W_t H_t^P + T_t^P \tag{4.20b}$$

$$\text{Wealthy Labour Supply:} \quad (1 - \tau_h)W_t = \chi \frac{C_t^W}{(1 - H_t^W)} \tag{4.20c}$$

$$\text{Wealthy Euler Equation:} \quad \frac{1}{C_t^W} = \beta E \left\{ (1 + (1 - \tau_K)R_t - \delta)\frac{1}{C_{t+1}^W} \right\} \tag{4.20d}$$

$$\text{Wealthy Budget Constraint:} \quad C_t^W + K_{t+1}^W = (1 - \tau_H)W_t H_t^W \tag{4.20e}$$

$$+ (1 + R_t(1 - \tau_K) - \delta)K_t^W \tag{4.20f}$$

$$\text{Production Function:} \quad Y_t = (e^{a_t}H_t)^\alpha K_t^{1-\alpha} \tag{4.20g}$$

$$\text{Wage Rate:} \quad W_t = \alpha(e^{a_t})^\alpha \left(\frac{H_t}{K_t}\right)^{\alpha-1} \tag{4.20h}$$

$$\text{Rental Rate:} \quad R_t = (1 - \alpha)(e^{a_t})^\alpha \left(\frac{H_t}{K_t}\right)^\alpha \tag{4.20i}$$

$$\text{Government Budget Constraint:} \quad e^{g_t}G_t + T_t = \tau_H W_t H_t + \tau_K R_t K_t \tag{4.20j}$$

4.2 Fiscal Policy

A competitive equilibrium of government policy, allocations and prices is defined such that the following conditions hold:

1. Households maximise their utility (4.2 and 4.7) subject to 4.3 and 4.8 such that 4.5, 4.6 and 4.9 hold.

2. Firms maximise profits such that 4.11 and 4.12 hold.

3. The government budget is balanced (4.17).

The government solves the Ramsey problem by finding allocations, prices and policy that maximise the social welfare function in 4.21. The government sets fiscal policy $\pi = \left\{ \tau_K, \tau_H, T \right\}_{t=0}^\infty$ under commitment in knowledge of firm and household behaviour, in effect setting all policy π, prices $p = \left\{ R, W \right\}_{t=0}^\infty$ and allocations $x = \left\{ H^W, H^P, C^W, C^P, K^W \right\}_{t=0}^\infty$.

$$E_0 \sum_{t=0}^\infty V_t = E_0 \sum_{t=0}^\infty \left[\gamma \left(\ln(C_t^P) + \chi \ln(1 - H_t^P) \right) + (1 - \gamma) \left(\ln \left(C_t^W \right) + \chi \ln \left(1 - H_t^W \right) \right) \right] \tag{4.21}$$

The parameter $0 < \gamma < 1$ indicates the government favouritism towards the wealthy or poor agents. When γ is close to one, the government is highly favourable towards the poor. When γ is close to zero, the government is highly favourable towards the wealthy. The Ramsey problem in Lagrangian form is shown in 4.22 where the policy choice is constrained by the household first order conditions, the household budget constraints and the government budget constraint.

$$
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{
\begin{array}{c}
V_t(U_t^W, U_t^P) \\
+ \underbrace{\varphi_t \left(F(K_t, H_t) - (1 - \tau_{K,t}) R_t K_t - (1 - \tau_{H,t}) W_t H_t - e^{g_t} G_t - T_t \right)}_{\text{Government Budget Constraint}} \\
+ \underbrace{\lambda_t^P \left((1 - \tau_H) W_t H_t^P + T_t^P - C_t^P \right)}_{\text{Poor Budget Constraint}} \\
+ \underbrace{\lambda_t^W \left(C_t^W + K_{t+1}^W = (1 - \tau_H) W_t H_t^W + (1 + R_t(1 - \tau_K) - \delta) K_t^W \right)}_{\text{Wealthy Budget Constraint}} \\
+ \underbrace{\phi_t^P \left(U_{H,t}^P - U_{C,t}^P (1 - \tau_{H,t}) W_t \right)}_{\text{Poor Labour Supply}} \\
+ \underbrace{\phi_t^W \left(U_{H,t}^W - U_{C,t}^W (1 - \tau_{H,t}) W_t \right)}_{\text{Wealthy Labour Supply}} \\
+ \underbrace{\psi_t \left(U_{C,t}^W - \beta U_{C,t+1}^W ((1 - \tau_{K,t}) R_t + 1 - \delta) \right)}_{\text{Euler Equation}}
\end{array}
\right\}
$$

$$(4.22)$$

It is not necessary to include the resource constraint (equation 4.19) as the government and household budget constraints holding imply that the resource constraint holds. The government budget constraint can be expressed in terms of the agent allocations as

$$
\begin{aligned}
F\left(K_t^W, H_t^W, H_t^P\right) &- (1 - \tau_{K,t}) R_t(1 - \theta) K_t^W \\
&- (1 - \tau_{H,t}) W_t \left(\theta H_t^P + (1 - \theta) H_t^W\right) - e^{g_t} G_t - T_t
\end{aligned}
$$

$$(4.23)$$

Solving the first order conditions for this Lagrangian for $t = 0$ and $t \geq 1$ lead to equations as outlined in C.4 in appendix C. Similarly to the representative agent model, solving the the first order conditions leads to an optimal capital tax of zero. The first order condition when $t > 0$ with respect to K_{t+1} (equation C.4k) can be written in steady state as

$$
\beta \left(\varphi((1 - \theta)R - (1 - \theta)(1 - \tau_K)R) + \lambda^W (1 + (1 - \tau_K)R - \delta) \right) = \lambda^W.
$$

$$(4.24)$$

Substituting in the steady state Euler equation we can find

$$\varphi \left((1 - \theta)R - (1 - \theta)(1 - \tau_K)R\right) = 0 \qquad (4.25)$$

which leads to

$$\tau_K = 0. \qquad (4.26)$$

This solution shows that in steady state, optimal tax on capital should equal zero.

4.3 Model Calibration and Dynare Setup

The model was calibrated as before (see table 1) with the added parameter of the share of *rule-of-thumb* agents. The value used was that as was estimated by Mankiw (2000) as was also used as the baseline value by Galí et al. (2007). This is shown in table 2.

Table 2: Heterogeneous Model parameter calibrations

Parameter	Value	Description
θ	0.5	Population share of *rule-of-thumb* agents

Two models were set up in dynare. The initial model had the labour tax treated as exogenous. This allowed the analysis of the distributional and welfare effects of tax reform. The dynare code for this model can be found in appendix D. The second model used the dynare Ramsey policy function. With this function, the model is set up with the equilibrium conditions as in equations 4.20. When dynare solves the model, finds the steady state and calculates the response to the shocks, it optimises the allocations, prices and policy is discussed above for a stated objective function. This is the social welfare function in equation 4.21. The dynare code for this model can be found in appendix E.

5 Results and Analysis

The optimal tax on capital has been shown analytically to be equal to zero in steady state. Running the model in dynare confirmed this solution with all government expenditure financed solely from taxing labour. Changing the government preferences did affect fiscal policy as shown in figure 5. The government becoming more favourable

towards the poor caused an increase in transfers to these agents. This was financed mainly by taxing labour but in part by increasing taxes on capital. It should be noted that the increases were very small at close to zero up until around $\gamma = 0.7$. When
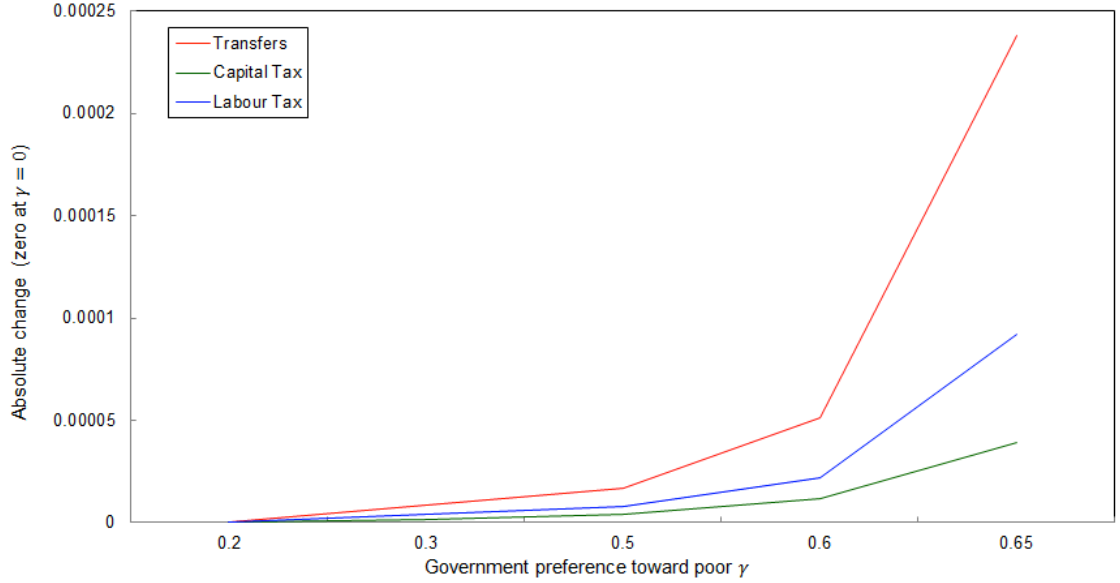

Figure 5: Effect of changing government preference toward poor γ on fiscal policy

the government preference was very high towards the poor, the transfers increased significantly so that *rule-of-thumb* agent consumption was almost entirely financed from government transfers. The transfers were financed by imposing a 96% tax on labour and a 2.7% tax on capital. Consequently the poor agent only provided a small amount of labour and the wealthy agent slightly reduced their labour supply. With reduced capital stock and labour supply, there were significant aggregate welfare losses from such a policy. It should be mentioned that with this setting the dynare model became unstable.

Clearly, even in the case of a government very biased towards the *rule-of-thumb* agents, the optimal tax rate on capital should be close to zero. If government preference is held balanced equally between all agents, higher government spending is financed solely from increases to labour tax. As this tax rate nears 100% the model understandably becomes unstable. Comparison of the steady state results between the models, as shown in table 3 confirm that the aggregate values are approximately equal regardless of the distribution of wealth.

The results that recommend a zero tax on capital are only truly relevant in the unre-

Table 3: Steady State Results with $\gamma = 0.5$: Model comparison

Variable	Representative Model	Heterogeneous Model
Output Y	1.56218	1.56217
Aggregate Consumption C	0.594196	0.594197
Aggregate Hours H	0.493051	0.493045
Aggregate Investment I	0.467986	0.467986
Aggregate Capital Stock K	20.3472	20.3472
Wage Rate W	2.1862	2.1862
Rental Rate R	0.0238006	0.0238011
Tax on Labour τ_H	0.463863	0.463878
Tax on Capital τ_K	-6.79×10^{-9}	3.91×10^{-6}
Government Transfers T	-	1.80×10^{-5}
Wealthy Capital Stock K^W	-	40.6944
Wealthy Hours H^H	-	0.486096
Wealthy Consumption C^W	-	0.602337
Poor Hours H^P	-	0.499983
Poor Consumption C^P	-	0.586058

alistic long run steady state. Further analysis was conducted to determine how fiscal policy should be conducted in the presence of economic fluctuations. The response of aggregate variables following shocks were generally as would be expected although the existence of *rule-of-thumb* agents did affect how taxes should be set over the business cycle.

5.1   Demand shock

Following a shock to government spending, the response of variables are as would be expected such as a drop in consumption and wage and an increase in rental rate and hours worked. However, there are two significant differences between the benchmark model and that with the second agent type. The fluctuations following the shock are much more subdued in the heterogenous model and there is a slower response so that the peak of the fluctuations occur several periods after the shock. In the representative agent model, the fluctuations peak immediately following the shock. The response of the wage and rental rates to a demand shock from both models are shown in figure 6.
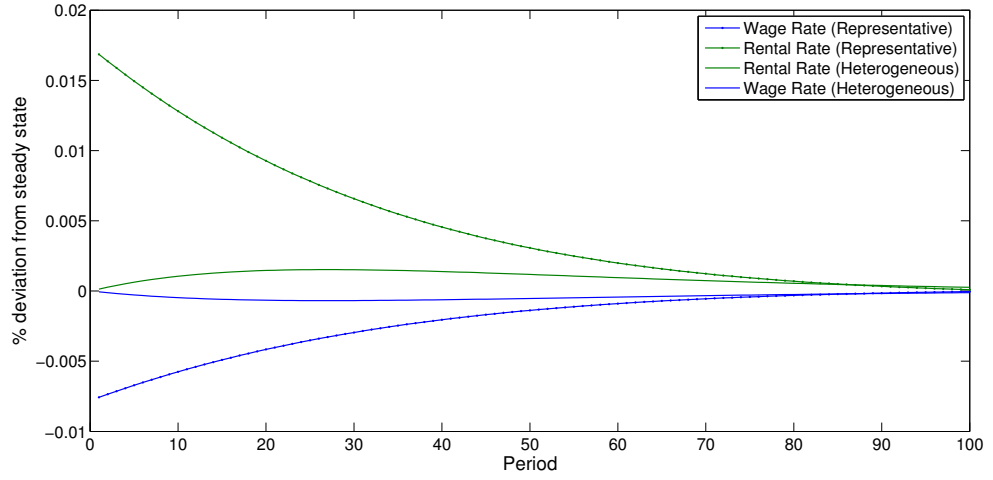
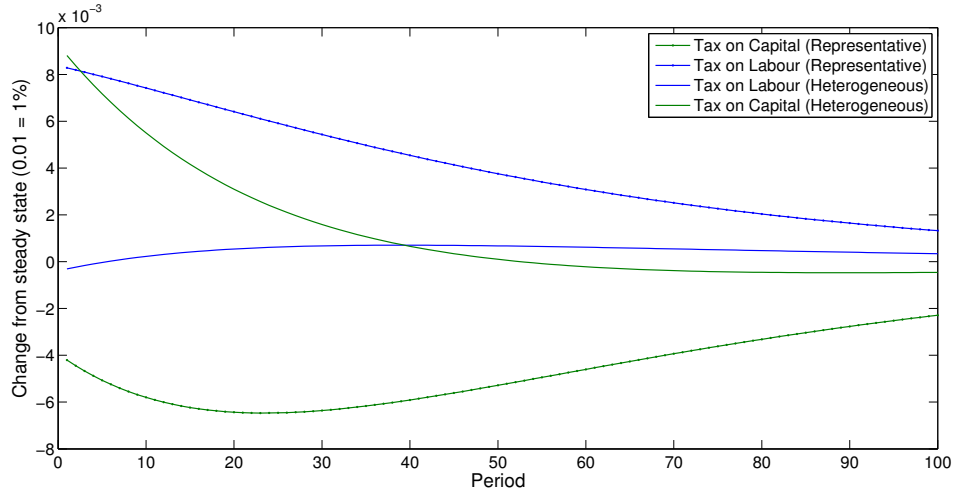**Figure 6:** Wage and rental rates following government spending shock e^{g_t}

Figure 7: Tax rates following government spending shock e^{g_t}

The second key difference is in the the optimal response of taxes on capital and labour as shown in figure 7. Introducing the second type of agent leads to interesting effects on these rates; to some extent the curvature of the tax rates is retained as they return to the steady state rates following the shock but the labour tax curve is shifted down whilst the capital tax is shifted up. In the representative model, the government should introduce a small subsidy on capital and increase labour taxes slightly to raise revenue for the additional spending. In the heterogeneous model, the government should finance the majority of the spending by taxing capital. This difference between the two models

is interesting as the changes in the optimal taxes in the benchmark model were explained by additional labour supply and a drop in the capital stock. These changes are also present in the heterogeneous model although the movement from steady state is much smaller.

Following the spending shock, the fall in consumption is similar between the two types of agent, with the drop in the consumption of the wealthy agents slightly greater than that of the poor. This is shown in figure 8. The labour supplied by the poor increases
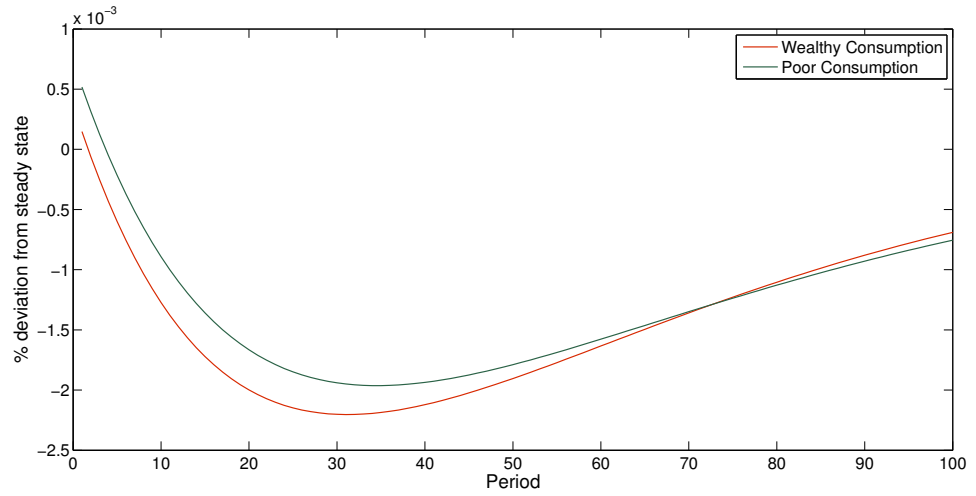
Figure 8: Consumption following government spending shock e^{g_t}

but only by a very small amount whilst the increase in the aggregate labour supply is mainly carried by the wealthy agents, as shown in figure 9. This is likely due to the drop in capital stock and a need for the wealthy agents to make up for lost income. This perhaps suggests the reason for the difference in tax rates between the two models. The demand shock causes a drop in consumption and investment and the government applies short run additional taxes to capital as the wealthy agents are able to supply more labour to help maintain income, the poor agents are unable to make up the income any other way. The model returns to the long run steady state of zero capital tax soon after. Indeed the capital tax drops below the steady state rate before returning to the steady state.

There are a couple of noteworthy comments to make. The obvious policy implication of this could be that it is optimal to finance extra spending with increased taxes on capital. Further analysis however calls this recommendation into question. If nominal
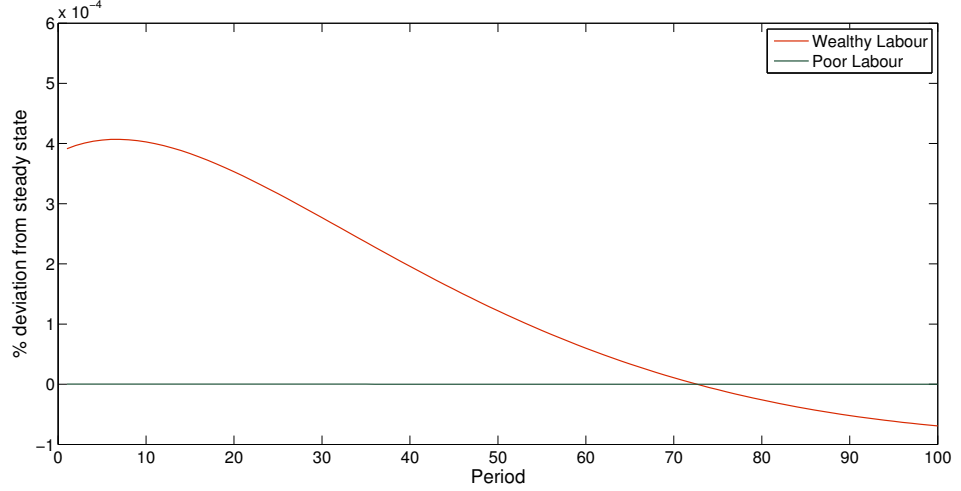
20

Figure 9: Labour supply following government spending shock e^{g_t}

rigidities were introduced into the model, as are in the majority of models used in policy analysis, the response of some of the key variables are likely to be different. This difference is more pronounced in the first few periods following a shock as it takes time for prices and wages to adjust. Rather than a fall in wage, many New Keynesian models predict an increase. Moreover, the increase in real interest rate, or the rental rate, can be far less pronounced.

Consequently it is arguable that introducing nominal rigidities into the model might change the dynamics and so it is possible that the tax on capital might not increase much, if at all. For this reason, it would be necessary to incorporate these features in order to surmise any policy recommendations on this issue.

5.2 Supply Shock

As with the benchmark model, the shock to labour productivity has a positive effect on output, consumption and investment as well as leading to an increase in wage and labour supply. Output increases for given factors of production leading to a household wealth effect. Introducing the second type of agent has the same interesting effect as with the previous shock in that despite the steady state values of the aggregate variables being the same between the models, applying the same shocks cause a much smaller reaction from most of the variables in the heterogeneous agent model. This is shown in the context of a supply shock in figure 10. The difference is most pronounced on the
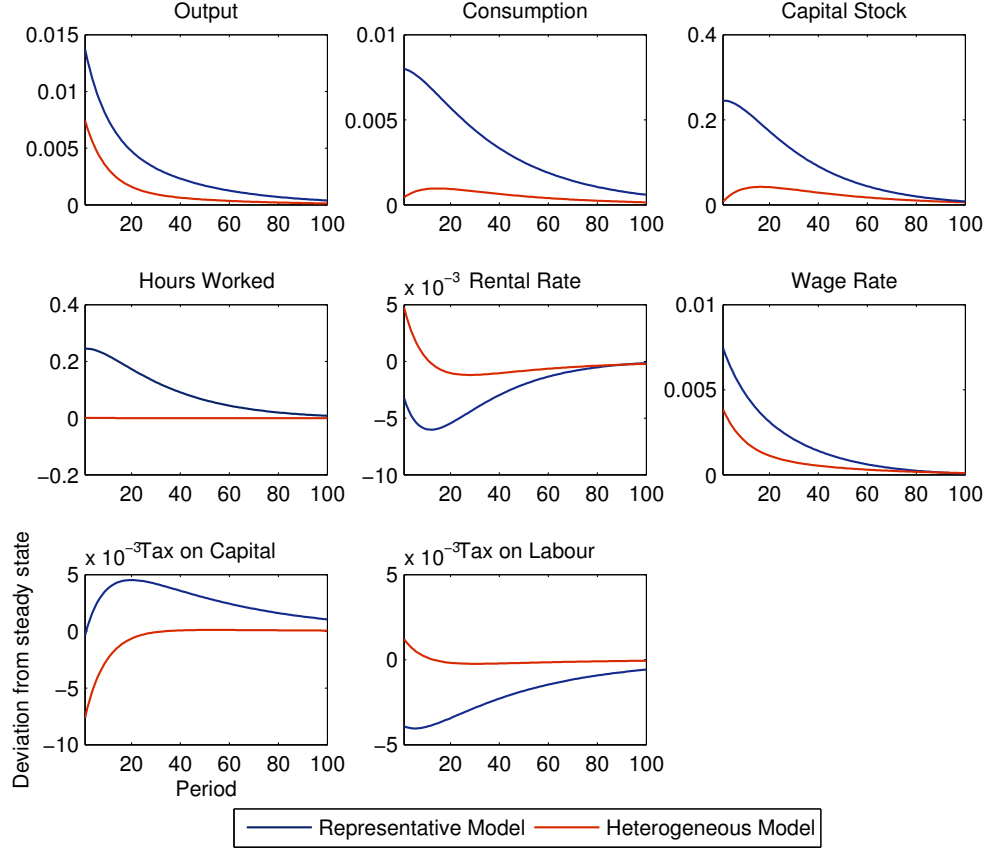
21

Figure 10: Aggregate impulse response functions following productivity shock e^{a_t}

labour supply which only experiences a very small rise and on the rental rate which whilst falling following the shock in the representative agent model, increases in the heterogeneous agent model. The optimal response of the two tax rates also differ.

In the representative agent model, increased investment results in a larger capital stock which leads to a drop in rental rate. With only half the number of savers in the heterogeneous agent model, there is greatly reduced investment increase following a smaller rise in output which leads to a much smaller rise in the capital stock. Consequently, the effect of the output increase on the rental rate is greater than that of the increased capital stock and so the rental rate rises rather than drops following the shock.

As can be seen in figure 10, the difference between the models has a significant effect on the optimal response of tax rates. Whilst in the representative model, the government applies a tax to capital and reduces the labour tax, the government in the heterogeneous
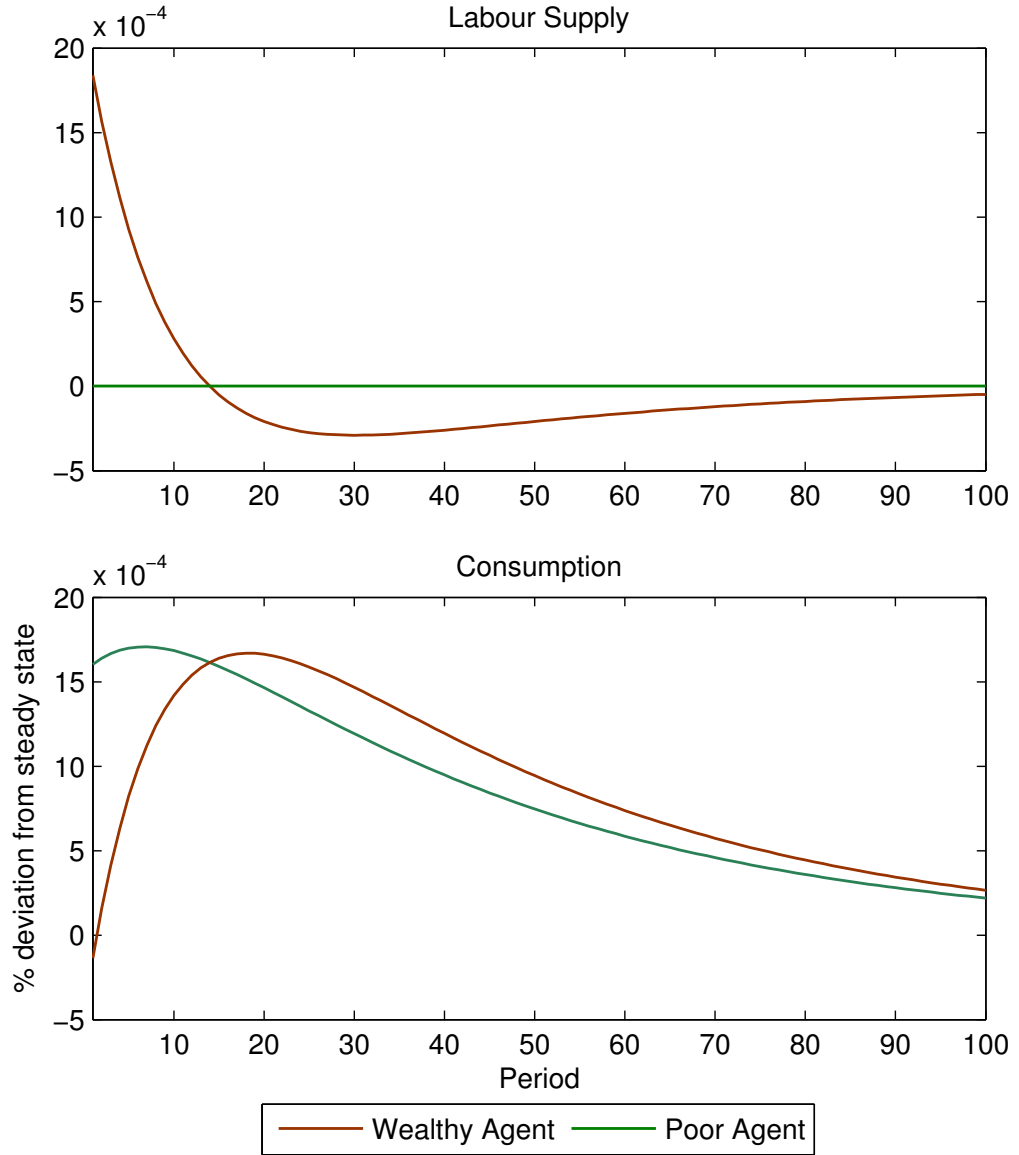
22

Figure 11: Impulse response functions of consumption and labour supply following productivity shock e^{a_t}

agent model apply a small subsidy to capital and raise labour taxes slightly. With a very small initial increase to the rate of investment, the government applies the subsidy to provide an incentive to the wealthy agents to save more and so increase the capital stock.

Figure 11 shows the agents reaction following the shock. The wealthy agents increase

the supply of labour as do the poor agents although only be a small amount. There is a similar response in consumption of the two agent types. The poor agents' consumption increases immediately following the shock whilst the wealthy agents' take longer to increase. This is likely due to the wealthy agents saving more initially using most of the additional income from the rise in hours worked to invest in the capital stock.

There are a few points worth making. As before, the constraint of the balanced government budget means that the taxes must respond to ensure that this requirement is kept. If this assumption were relaxed it is likely to have an effect on optimal fiscal policy following the shock. Similarly to the suggestion before, the introduction of nominal rigidities may have an effect on the reaction of the key variables and indeed the optimal response of fiscal policy. Another feature is that whilst the labour supplied by the poor agents is affected by economic fluctuations it is only by a tiny amount in comparison to that supplied by the wealthy agents. Despite this difference, the change to consumption is similar; the wealthy agents work more hours in order to increase the savings rate. This leads to higher consumption in the periods further along after the shock.

5.3 Government Bias

Varying the government preference towards either the rich or the poor does not significantly alter the optimal values of the variables in steady state. As mentioned above and demonstrated in figure 5, as the government becomes more favourable towards the poor, transfers to the poor agent rise financed by a mixture of both capital and labour taxes. This, however, is by an extremely small amount up to around $\gamma = 0.65$. Above this point, the model becomes unstable and the results are unreliable. The model remains stable with very low values of γ and the values of the steady state variables remain consistent. It is possible that as the preference becomes very strong in favour of the poor, there is political pressure to increase government transfers financed by taxes on capital. This is a non-optimal policy and causes the model to fail. What is striking from the evidence, is that the model indicates that a policy that is optimal for the wealthy is also optimal for the poor. However the presence of inequality within the model is necessary to propose optimal fiscal policy.

When considering optimal policy over the business cycle, there is little change to the optimal response. Figures 12 and 13 depict the impulse response functions of the
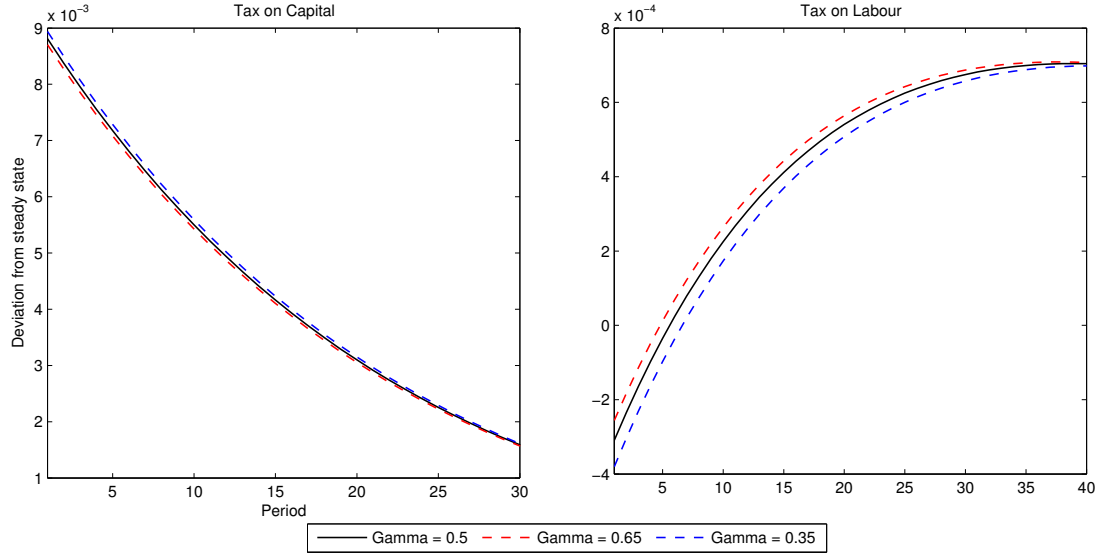
24

Figure 12: Tax impulse response functions following spending shock e^{g_t}: Varying γ

optimal tax rates as γ varies from a government biased towards the poor, a balanced government with no bias and a government biased towards the wealthy. As can be seen

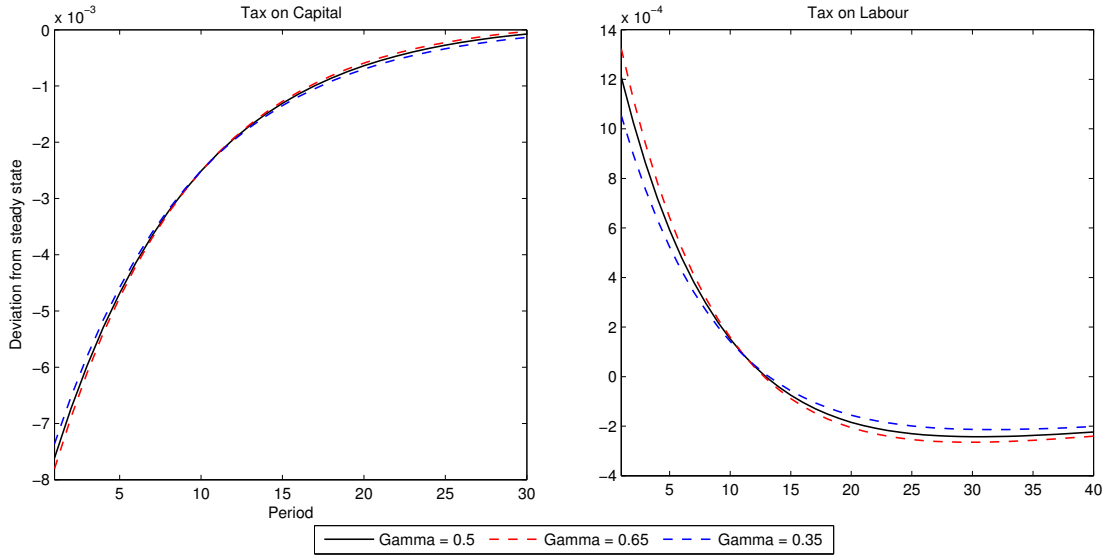**Figure 13:** Tax impulse response functions following productivity shock e^{a_t}: Varying γ

from the plots, the optimal response is very similar regardless of the government bias. Most aggregate variables are identical for the different values of γ, the exception being the labour supply. As the tax rates differ slightly, the hours worked by both agents also differ for different government preferences.

In order to provide a better analysis of varying government bias, incorporating market incompleteness such as information asymmetries may well cause the reactions of agents to differ. This may in turn lead to different optimal policy responses.

5.4   Tax Reform

Having focused on optimal fiscal policy, attention turns to the process of changing the composition of labour and capital taxes. Domeij & Heathcote (2004) argue that changing the balance of these taxes is likely to incur significant distributional changes. The same model with inequality is used to analyse the effect of tax reform.

In order to run this model in dynare, the model was calibrated with the tax rates in force in the United Kingdom. The income tax was set to the basic rate of 20% (HM Revenue & Customs 2012*b*) and the capital tax was set to 18% as the 2011-2012 capital gains tax rate (HM Revenue & Customs 2012*a*). The government spending was then calibrated at that necessary to run the model with these tax rates. The model was tested to see what effect changing the composition of these tax rates had to finance the same level of public spending.

5.4.1   Increase in Labour Tax with Reduction in Capital Tax

The plot shown in figure 14 shows the deviation of the agents' period utility following a 5% permanent increase in labour tax. The tax rate of capital reduces to less than 3%. As can be seen, the utility of both types of agent drops immediately following the change in tax rates. As the tax rates are moving towards those at which the model optimises, there is an aggregate welfare gain in the long run. Figure 14 shows that utility for both agent types is greater in the new steady state and whilst the difference is not great, there is a distributional impact of changing the composition of taxes in this way as the welfare gains of the wealthy are greater than those of the poor. The wealthy also have a much greater initial drop in welfare following the tax rate change.
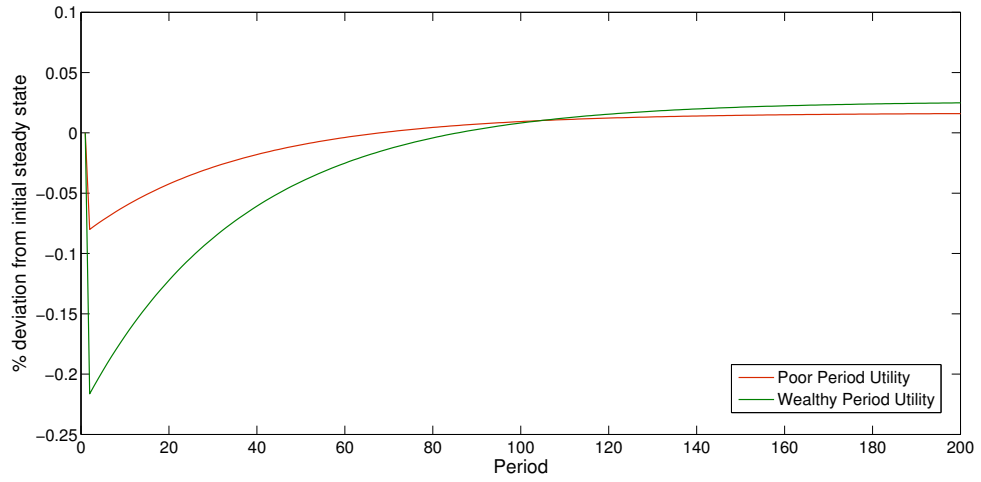
Figure 14: Agent period utility following 5% increase in labour tax τ_H

5.4.2 Reduction in Labour Tax with Increase in Capital Tax

The plot shown in figure 15 shows the deviation of agent period utility following a 5% permanent decrease in labour tax. The tax rate of capital increases to over 37%. Following the change to the composition of tax rates, the utility for both types of agent
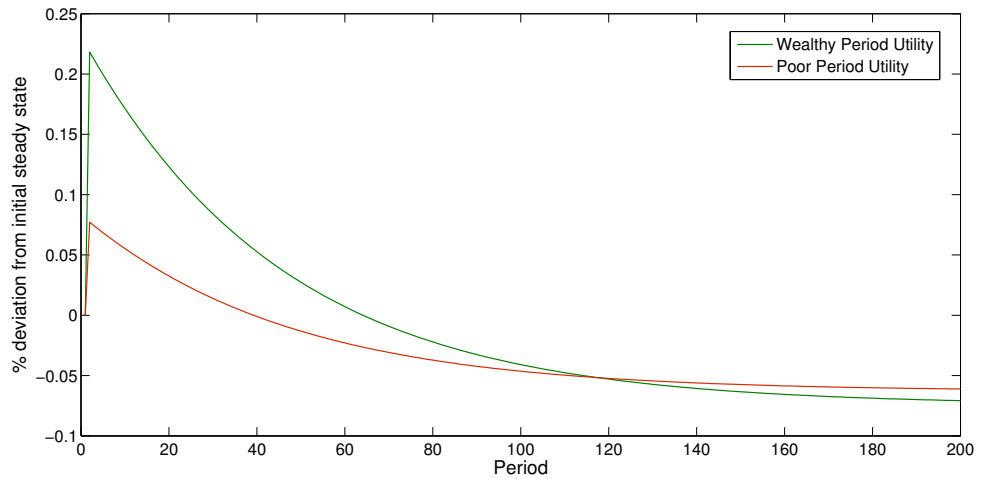

Figure 15: Agent period utility following 5% decrease in labour tax τ_H

experiences a large increase, particularly for the wealthy agents. As a mirror image to the labour tax increase, the model moves away from the the tax rates at which the government optimises and so there is a long run reduction in social welfare. Again there

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is a distributional dimension as the wealthy agents face a greater welfare loss than do the poor agents.

Despite the simplicity of the model, the analysis to some extent has confirmed the conclusion of Domeij & Heathcote (2004) who find that whilst long run welfare can be expected to increase following a reduction of capital tax, the short run cost from the increased labour taxes are too heavy a price to pay. This analysis has shown that immediately following the change in tax rates there are high losses in welfare and agent welfare only gradually returns to a steady state value slightly higher than the initial value. Conversely, increasing the tax rate on capital provides a short run increase to welfare.

6 Conclusions

The central conclusion of the analysis is that whilst the presence of agent heterogeneity does not affect the optimal steady state behaviour of aggregate variables and fiscal policy, it is necessary to understand both the response of key variables to economic fluctuations and how fiscal policy should be set over the business cycle. It was also noticed throughout the study that the level of assumptions such as balanced government budget, exogenous government spending, perfect competition, no nominal rigidities etc, caused limitation to the analysis and left plenty of room to build on the model. Some of these are discussed in section 6.1.

The long run optimal tax rate on capital was found to be zero regardless of the government bias. As such, all government spending should be financed with revenue from a tax on labour. It is quite possible that in the presence of incomplete markets (such as Aiyagari 1995) that taxes on capital would be optimised above zero. Government transfers to the poor agent were also found to be close to zero in this model, even over the business cycle. If a more realistic distribution of income were modelled, it is possible transfers would be a useful tool for re-distribution. In this model, wage level was flat and labour income financed the majority of consumption. With only a small difference between the two types of agent there was no significant role for redistributive policy.

Whilst introducing heterogeneity alone was not enough to change the steady state optimal tax on capital, it did have a significant effect on how fiscal policy should be set

over the business cycle. It was found that temporary increases to government spending should be financed with temporary increases to capital tax rates. This was opposite to the recommendation by the representative agent model. Following the productivity shock, it was optimal to apply additional tax on labour to finance a subsidy on capital. Again, this was in conflict with the representative agent model which recommended a temporary capital tax to finance a small drop in labour tax.

With this recommendation it is necessary to make several points. Firstly, and as mentioned previously, the presence of nominal rigidities are likely to affect the behaviour of variables and hence optimal fiscal policy significantly following shocks. In a typical New Keynesian model for instance, wages are sluggish to respond to shocks and so the behaviour of labour supply will be affected. The real interest rate, or rental rate, is also likely to be affected by nominal frictions. Consequently the timing and level of tax rate changes over the business cycle are likely to optimise differently with these frictions present.

Secondly, the model seems to be contrary to other studies (e.g. Giavazzi & McMahon 2012) that show that lower income households tend to respond more to fluctuations by cutting consumption and working more hours following a government spending shock for instance. This analysis suggests that the wealthy households react more to fluctuations. By endogenising the agent heterogeneity and introducing information asymmetries for example, it is quite possible the model would better predict the heterogeneous response to shocks and any distribution issues that this might lead to.

The final point is that in attempting to make policy recommendations by studying the impact of shocks, the analysis enters the debate on the source of economic fluctuations. Some (e.g. Christiano, Eichenbaum & Vigfusson 2003) argue that productivity shocks are largely responsible for generating business cycles whilst others (e.g. Galí & Rabanal 2004) argue that productivity shocks plays a minor role and fluctuations in aggregate demand play a greater role. How this is interpreted has important policy implications in how the government should set fiscal policy over the business cycle. More analysis modeling agent heterogeneity are likely to significantly add to this debate.

To some extent, the analysis confirmed the conclusion of Domeij & Heathcote (2004) regarding tax reforms. Domeij & Heathcote found that although there could be welfare gains in reducing taxes on capital, the heavy short run welfare costs mean that the reform would be too costly. This study found that reducing tax on capital would take

the economy towards optimal policy, allocations and prices and hence a higher long run level of welfare; in the short run there are significant welfare costs. Conversely the opposite is true for increases to capital tax. There were also distributional effects. As the capital tax was reduced, the wealthy agents gained more than the poor agents in the long run. Again, if other frictions were introduced, the distributional effects may well be greater suggesting that conducting the analysis with the presence of such features would produce results of interest.

6.1 Considerations and Further Research

There are some notable absentees in the analysis, such as government debt and unemployment. These will be discussed briefly here as possible extensions to the study. Perhaps the most important next step is to take the model to the data. By estimating the model, the dynamics of heterogeneous variables would be better interpreted and so would provide richer fiscal policy recommendations. The empirics would inform the theory which in turn helps reinterpret the empirics. In the absence of empirics, the analysis become more of an academic process.

6.1.1 Involuntary Unemployment

In the proposal for this study I stated that any model seriously attempting to suggest welfare optimal policy must have a theory of unemployment. Due to the constraints inherent to a project of this nature, unemployment was not included in the analysis. Incorporating this important feature into the model would be a natural next step. Blanchard (2009) suggested that the welfare costs of economic fluctuations tend to affect the unemployed disproportionately. Involuntary unemployment is at significant levels across the western economies, particularly amongst young people and so it is vital that the role government policy can play in this issue is well understood.

There have been some recent attempts to build realistic models incorporating involuntary unemployment such as Galí, Smets & Wouters (2011) and Christiano, Trabandt & Walentin (2010). In doing so they have been able to accurately estimate the response of unemployment to shocks. Michaillat (2012) provides evidence that fiscal policy in the form of spending on public sector jobs can provide an effective way to reduce unemployment.

6.1.2 Government Debt

The analysis was conducted with the unrealistic constraint of a balanced government budget. There are several reasons why it would be beneficial to relax this assumption to investigate both optimal debt levels and the topical issue of debt consolidation. Firstly, the optimal tax rates following shocks in this study are likely to be different when optimal debt is introduced. Further analysis in the presence of government debt would be of interest to find optimal tax rates together with optimal debt over the business cycle. Secondly, as Kumhof & Yakadina (2010) demonstrate, there is a disjuncture between the theoretical optimal government debt (usually negative) and the optimal empirical debt levels (usually positive). Attempting to reconcile this in the context of an heterogeneous economy would be worthwhile so to understand the distributional impact and whether this effects optimal debt levels.

Finally, it would be of interest due to the current political fallout and debate around the issue of austerity during periods of sluggish growth. Bi, Leeper & Leith (2012) discuss the composition of fiscal consolidation including the duration, the monetary policy stance, the level of government debt in determining whether a policy is expansionary and successful in stabilising levels of government debt. I would also argue for analysis into the effects on different types of household. It has been argued that the costs of fiscal consolidation fall disproportionately on the poor; using a panel of 18 industrialised economies, Agnello & Sousa (2011) demonstrate that inequality consistently increases during times of fiscal consolidation. This was shown to be more significant when spending cuts rather than tax increases are used. In addition to understanding whether certain policies are indeed stabilising and expansionary, further analysis of this issue would perhaps go some way to answering the question as to whether any costs to aggregate welfare are worthwhile in light of solving welfare costs on the lower income households.

6.1.3 Tax progressivity

Another assumption that limits the scope of the paper is that all agents in the model have equal labour productivity. As such, there is no wage rate difference between wealthy agents and poor agents and so tax on labour is at a flat rate. Although this simplifies the model, it limits the scope to only focus on the composition of tax rates on capital and labour. On the important and topical issue of optimal income tax

progressivity, the analysis is silent. It would be a good extension to the project to incorporate a realistic distribution of labour type and productivity and find an optimal progressive tax code. Varying the labour productivity would also lead to a more realistic income distribution. Having a single wage rate causes the wealthy agents, with an additional source of income, to always work slightly less and consume slightly more than the poor agents.

This issue is often one for debate, particularly surrounding the release of the annual government budget in March, with the focus usually on the marginal tax rates. There is a strong argument that a high top-band income tax rate has a disincentive effect on economic growth (Meltzer & Richard (1981) for example). A recent study however (Roine & Waldenström 2008), although not their central thesis, seems to suggest that the wealthy get wealthier regardless of the progressivity of the tax code. In his blog, Thoma (2012) suggests that in this case, high top-band income tax rates have a far smaller disincentive effect than has been previously supposed. Romer & Romer (2012) back this up with an analysis into the incentive effects of marginal tax rates. Another study concludes that successful tax reform should both increase the average labour tax and increase the progressivity (Ábrahám & Cárceles-Poveda 2010).

6.1.4 Nominal Frictions

The model outlined in this paper provides no role for monetary policy with fiscal policy providing the only method of stabilization in addition to the distributional role. The analyses have suggested that the introduction of nominal rigidities may have a significant effect on the response of key variables following shocks, particularly in the initial periods. In light of this, there is sufficient rationale for including nominal rigidities in order to better represent the movement of variables over the business cycle and suggest optimal taxation. Monacelli & Perotti (2011) demonstrate that the presence of such rigidities are necessary to determine which type of agent bears the cost of fiscal stimulus as well as the size of the multiplier. This clearly recommends the incorporation of both nominal rigidities and heterogeneity in the analysis of fiscal policy.

6.1.5 Sources of Heterogeneity

The final point is a significant weakness in the analysis in that the *rule-of-thumb* behaviour is not endogenised. This causes the model to fall foul to the Lucas critique, as the model relies on statistical information showing that a portion of society exhibit this type of behaviour. The critique holds that as changing government policy may cause the agent behaviour to change, the model is not sufficient for policy evaluation.

To improve this, the model should include optimising agents that choose this behaviour due to information asymmetries or some other friction. Doing so would allow different policies to be tested and chosen with insight into how agent behaviour would be affected in a more sophisticated way. It would also benefit in helping to understand the empirics; why is this behaviour observed? Agent heterogeneity may also be caused by other factors such as varying labour productivity or non-identical utility functions.

One popular methodology in which heterogeneous behaviour is endogenised is by introducing uninsurable income shocks (e.g. Domeij & Heathcote 2004, Ábrahám & Cárceles-Poveda 2010). Ábrahám & Cárceles-Poveda argue that doing so allows an accurate representation of wealth distribution and provides models suitable for policy analysis. Together with the income shocks, the authors introduce the possibility of default on financial liabilities in which agents are excluded from future trading. These agents represent the *rule-of-thumb* consumers.

Despite the value of having microfoundations to the heterogeneous behaviour, the model outlined in this paper could be a good first step in policy evaluation. Rooting all relationships in optimising behaviour does not guarantee the reliability of a model and indeed there are examples of economic models constructed with microfoundations at the core but *ad hoc* observed relationships at the periphery[3]. Extending this project should however include endogenising the heterogeneous behaviour as realistically as possible.

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[3]Such as the Bank of England Quarterly model (Harrison, Nikolov, Quinn, Ramsay, Scott & Thomas 2005). Chapter three of Harrison et al. (2005) outlines the model core, chapter four discusses the non-core/hybrid core which includes *ad hoc* economic relationships.

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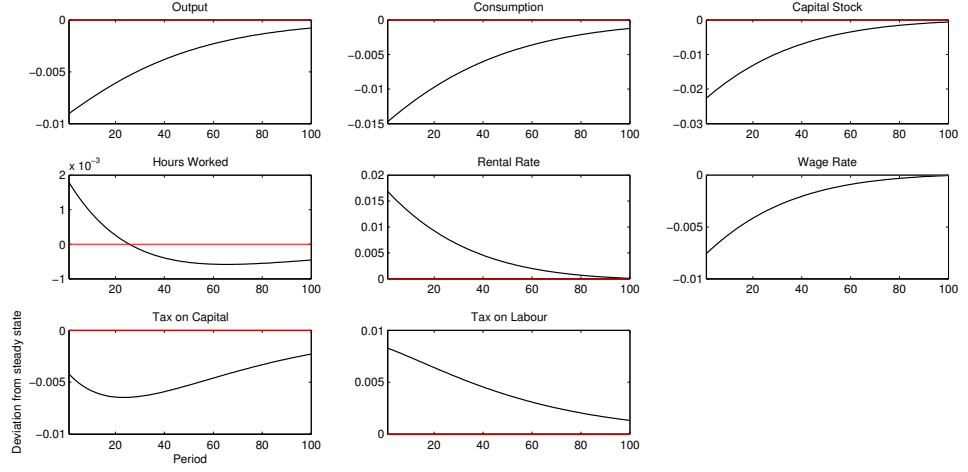# Appendix A Representative Model: Impulse Response Functions

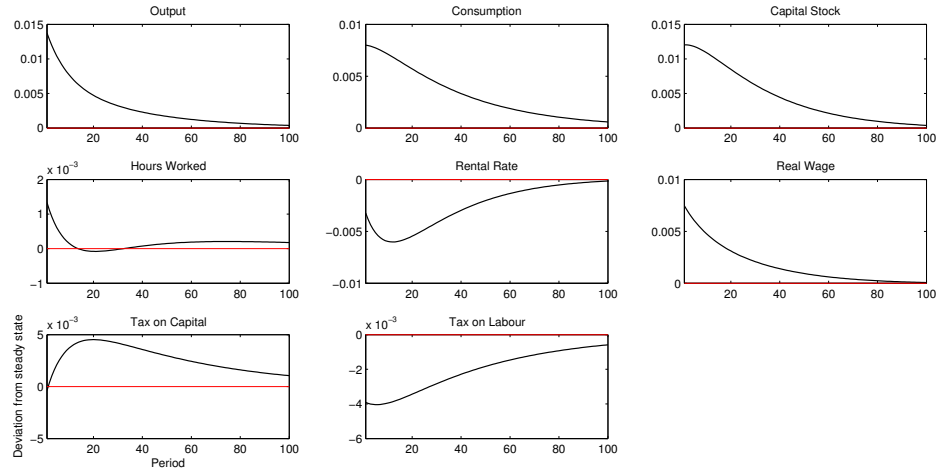**Figure 16:** Demand shock e^{g_t} impulse response function plots

Figure 17: Supply shock e^{a_t} impulse response function plots

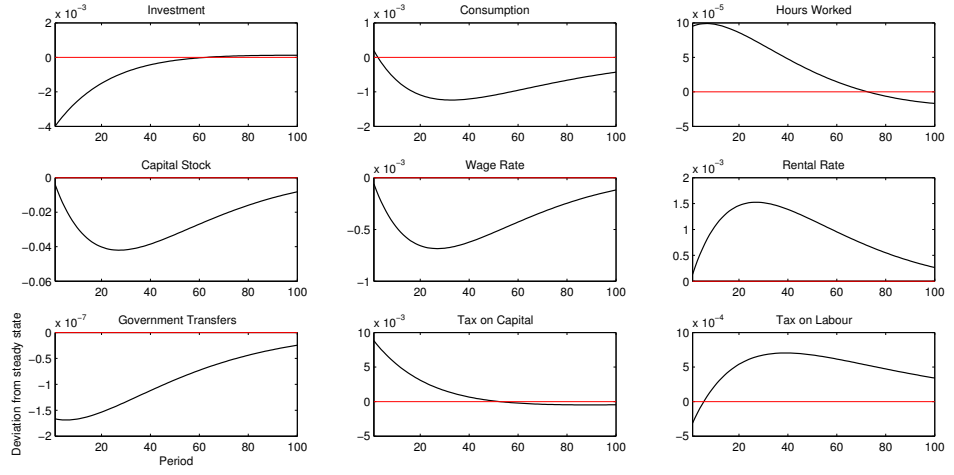# Appendix B   Heterogeneous Model: Impulse Response Functions

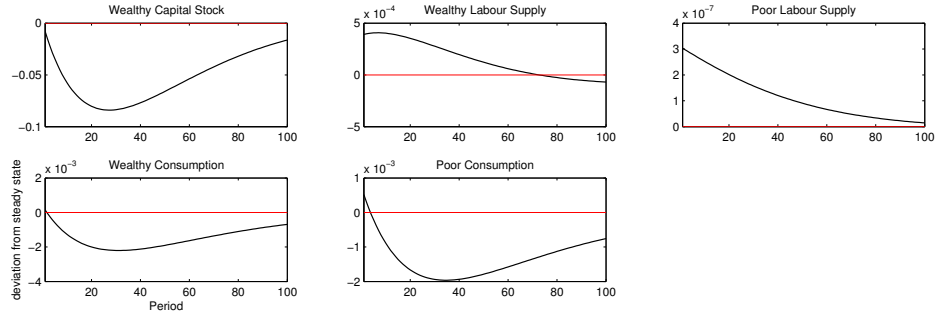**Figure 18:** Demand shock e^{g_t} impulse response functions of aggregate variables

Figure 19: Demand shock e^{g_t} impulse response functions of household variables
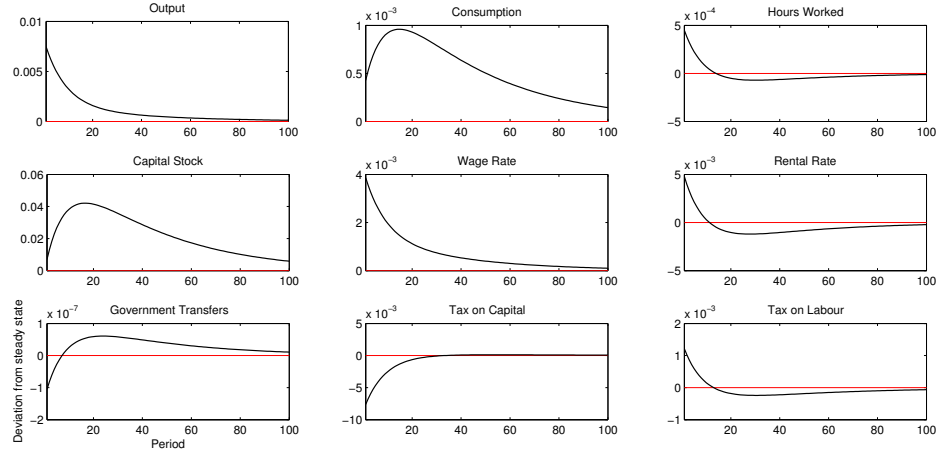
Figure 20: Supply shock e^{a_t} impulse response functions of aggregate variables
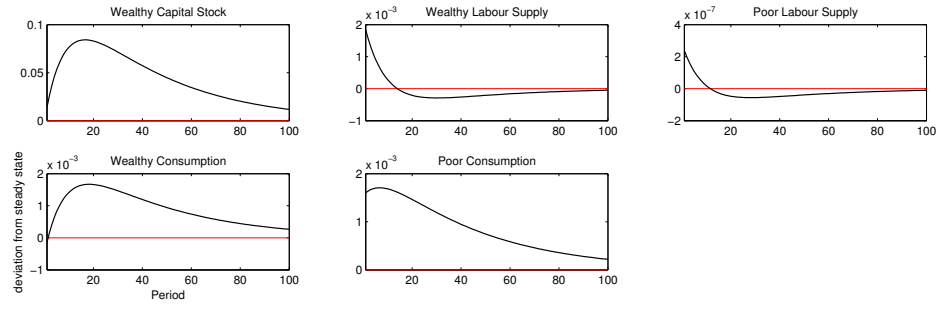

Figure 21: Supply shock e^{a_t} impulse response functions of household variables