Precautionary Saving over the Business Cycle*

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Abstract

We study the macroeconomic implications of time-varying precautionary saving within a general equilibrium model with borrowing constraint and both aggregate shocks and uninsurable idiosyncratic unemployement risk. Our framework generates limited cross-sectional household heterogeneity as an equilibrium outcome, thereby making it possible to analyse the role of precautionary saving over the business cycle in an analytically tractable way. The time-series behaviour of aggregate consumption generated by our model is much closer to the data than that implied by the comparable hand-to-mouth and representative-agent models, and comparable to that produced by the (intractable) Krusell-Smith (1998) model. (*JEL* E20, E21, E32)

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1 Introduction

Although precautionary saving against uninsurable income shocks has been widely analysed theoretically and empirically, it has remained difficult to incorporate into dynamic general equilibrium models for at least two reasons. First, the lack of full insurance usually produces a considerable amount of agent heterogeneity, essentially because the wealth of any particular agent –and, by way of consequence, the decisions it makes– generally depends on the entirely history of income shocks that this agent has faced (see, e.g., Huggett, 1993; Aiyagari, 1994; Guerrieri and Lorenzoni, 2011). Second, aggregate shocks turn the cross-sectional distribution of wealth into a time-varying state variable, the evolution of which every agent must forecast in order to make their best intertemporal decisions. In their pioneering contribution, Krusell and Smith (1998) have proposed a solution to this problem, which consists in simplifying the agents's forecasting problem by approximating the full cross-sectional distribution of wealth with a small number of moments. However, the lack of tractability of the underlying problem makes their solution method operational only in relatively simple environments; in particular, both the number of state variables and the support for the exogenous shocks must remain limited.[1]

In this paper, we construct a class of heterogenous-agent models with incomplete markets, borrowing constraints and both aggregate and idiosyncratic labour income shocks than can be solved under exact cross-household aggregation and rational expectations. More specifically, we exhibit a set of sufficient conditions about preferences and the tightness of the borrowing constraint, under which the model endogenously generates a cross-sectional distribution of wealth with a limited number of states; exact aggregation directly follows. This approach makes it possible to derive analytical results and incorporate time-varying precautionary saving into general equilibrium analysis using simple solution methods –including linearisation and undetermined coefficient methods. In particular, our analysis allows the derivation of a common asset-holding rule for employed households facing incomplete insurance, possibly expressed in *linear* form, which explicitly connects precautionary wealth accumulation to the risk of experiencing an unemployment spell.[2] Additionally, our model can be simulated

[1] Krusell et al. (2010, p. 1497) refer to this approach as one in which "consumers have boundedly rational perceptions of the evolution of the aggregate state". As mentioned there, the approach is valid under the conjecture that "approximate aggregation" holds, so that the forecasting rules used by the agents take the economy close to the true rational-expectations equilibrium. See Heathcote et al. (2010) for a discussion of this point and Algan et al. (2010) for a survey of alternative computational algorithms.

[2] Since a substantial fraction of the households does not achieve full self-insurance in equilibrium (despite

with several –and possibly imperfectly correlated– aggregate shocks with continuous support; we consider three such shocks in our baseline specification (i.e., technology, job-finding and job-separation shocks).

In order to isolate, both theoretically and quantitatively, the precautionary motive in the determination of households' savings, our general framework incorporates both patient, "permanent-income" consumers and impatient consumers who are imperfectly insured and may face occasionally binding borrowing constraints. Aside from the baseline precautionary-saving case just discussed, wherein impatient households hold a time-varying buffer-stock of wealth in excess of the borrowing limit, our framework also nests two special cases of interest: the *representative-agent* model and the *hand-to-mouth* model. The representative-agent model arises as a limit of our incomplete-market model when the economy becomes entirely populated by permanent-income consumers. The hand-to-mouth model –a situation when impatient households face a binding borrowing limit in every period– endogenously arises when the precautionary motive becomes too weak to offset impatience, so that impatient agents end up consuming their entire income in every period.[3] We trace back the strength of the precautionary motive –and thus whether or not impatient households are ultimately willing to save– analytically to the deep parameter of the model, most notably the extent of unemployment risk, the generosity of the unemployment insurance scheme, and the tightness of the borrowing constraint.

We then use our framework to identify and quantify the specific role of incomplete insurance and precautionary wealth accumulation –as opposed to, e.g., mere borrowing constraints– in determining the volatility of aggregate consumption and its co-movements with output. To this purpose, we calibrate the model so as to match the main features of the cross-sectional distributions of wealth and nondurables consumption in the US economy –in addition to matching other usual quantities. We then feed the calibrated model with aggregate shocks to productivity and labour market transition rates with magnitude and joint behaviour that are directly estimated from post-war US data. We find the time-series behaviour of aggregate

precautionary wealth accumulation), they experiences a discontinuous drop in income and consumption when unemployment strikes. This drop being of first-order magnitude, changes in the perceived likelihood that it will occur have a correspondingly first-order impact on the intensity of the precautionary motive for accumulating assets *ex ante.*

[3] In this case, our economy collapses to a two-agent one of the kind studied by, e.g., Becker and Foias (1987), Kiyotaki and Moore (1997), or Iacoviello (2005). We refer to this situation as the "hand-to-mouth" case even when the borrowing limit is not strictly zero –since agents then end up consuming their entire income, including their *negative* capital income.

consumption generated by our baseline precautionary-saving model to be much closer to the data than those implied by the comparable hand-to-mouth and representative-agent models. Perhaps unsurprisingly, the representative-agent limit of our framework generates to little consumption volatility and too low a consumption-output correlation. More interestingly, the comparable hand-to-mouth model generates not only too high a consumption-output correlation (due to constrained households' consumption tracking their income), but also *too little consumption volatility*. In contrast to the latter two models, the precautionary-saving model is able to generate a high volatility of aggregate consumption, because consumption then responds to *expected* labour-market conditions (via the precautionary motive) in addition to *current* labour market conditions (the key determinant of aggregate consumption in the hand-to-mouth case). Time-varying precautionary savings also contribute to relax the tight output-consumption association predicted by the pure hand-to-mouth model, but without taking it as low as in the representative-agent case. To complete the picture, we also compare the moments of interest implied by our baseline precautionary-saving model with those generated by the full-fledged heterogenous-agent model of Krusell and Smith (1998, Section IV). Despite their structural differences, the Krusell-Smith model and our baseline incomplete-market model predict similar levels of consumption volatility and consumption-output correlation, and those most in line with the data relative to the representative-agent and hand-to-mouth models.

Our analysis differs from earlier attempts at constructing tractable models with incomplete markets, which typically restrict the stochastic processes for the idiosyncratic shocks in ways that makes them ill-suited for the analysis of time-varying unemployment risk. For example, Constantinides and Duffie (1996) study the asset-pricing implications of an economy in which households are hit by repeated permanent income shocks. This approach has been generalised by Heathcote et al. (2008) to the case where households' income is affected by insurable transitory shocks, in addition to imperfectly insured permanent shocks. Toche (2005), and more recently Carroll and Toche (2011) explicitly solve for households' optimal asset-holding rule in a partial-equilibrium economy where they face the risk of permanently exiting the labour market. Guerrieri and Lorenzoni (2009) analyse precautionary saving behaviour in a model with trading frictions a la Lagos and Wright (2005), and show that agents' liquidity hoarding amplify the impact of i.i.d. (aggregate and idiosyncratic) productivity shocks. Relative to these models, ours allows for stochastic transitions across labour market statuses, which implies that individual income shocks are transitory (but persistent) and have a conditional distribution that depends on the aggregate state. The model is thus fully consistent with the flow approach to the labour market and can be evaluated using direct evidence on the cyclical movements in labour market transition rates.[4]

The remainder of the paper is organised as follows. The following section introduces the model. It starts by describing households' consumption-saving decisions in the face of idiosyncratic unemployment risk; it then spells out firms' optimality conditions and characterises the equilibrium. In Section 3, we introduce the parameter restrictions that make our model tractable by endogenously limiting the dimensionality of the cross-sectional distribution of wealth. Section 4 calibrates the model and compares its quantitative implications to the data, as well as to three alternative benchmarks –the representative-agent model, the hand-to-mouth model and the Krusell-Smith model. Section 6 concludes.

2 The model

The model features a closed economy populated by a continuum of households indexed by i and uniformly distributed along the unit interval, as well as a representative firm. All households rent out labour and capital to the firm, which latter produces the unique (all-purpose) good in the economy. Markets are competitive but there are frictions in the financial markets, as we describe further below.

2.1 Households

Every household i is endowed with one unit of labour, which is supplied inelastically to the representative firm if the household is employed.[5] All households are subject to idiosyncratic changes in their labour market status between "employment" and "unemployment". Employed households earn a competitive market wage (net of social contributions), while unemployed households earn a fixed unemployment benefit $\delta^i > 0$.[6]

[4] Carroll (1992), and more recently Parker and Preston (2005) have suggested that changes in precautionary wealth accumulation following (countercyclical) changes in the extent of unemployment risk significantly amplify aggregate consumption fluctuations. This motivates our focus on idiosyncratic and time-varying unemployment risk as a driver of aggregate savings, a focus that we share with Krusell and Smith (1998).

[5] Our model ignores changes in hours worked per employed workers, since those play a relatively minor role in the cyclical component of total hours in the US (see, e.g., Rogerson and Shimer, 2011). Incorporating an elastic labour supply for employed workers would be a simple extensions of our baseline specification.

[6] Following much of the heterogenous-agent literature, we focus on (un)employemnt risk as the main source of idiosyncratic income fluctuations at business-cycle frequency. Our model could easily be extended to introduce wage risk in addition to employment risk (as in, e.g., Low et al. (2010)).

We assume that households can be of two types, *impatient* and *patient*, with the former and the latter having subjective discount factors $\beta^I \in (0,1)$ and $\beta^P \in \left(\beta^I, 1\right)$ and occupying the subinterval $[0,\Omega]$ and $(\Omega,1]$, respectively, where $\Omega \in [0,1)$. While not necessary for the construction of our equilibrium with limited cross-sectional heterogeneity below, the introduction of patient households will allow us to generate a substantial degree of cross-sectional wealth dispersion, since patient households will end up holding a large fraction of total wealth in equilibrium.[7] However, in contrast to models featuring heterogenous discount factors wherein impatient households face a binding borrowing limit and hence behave like mere "hand-to-mouth" consumers[8], most of the impatient in our baseline model will hold a wealth buffer in excess of the borrowing limit –and will thus *not* face a binding constraint. As we shall see in Section 4 below, that these households do not behave as hand-to-mouth consumers crucially matters for the aggregate time-series properties of the model.

Unemployment risk. The unemployment risk faced by individual households is summarised by two probabilities: The probability that a household who is employed at date $t-1$ becomes unemployed at date t (the job-loss probability s_t), and the probability that a household who is unemployed at date $t-1$ stays so at date t (i.e., $1-f_t$, where f_t is the job-finding probability). The law of motion for employment is[9]

$$n_t = (1-n_{t-1}) f_t + (1-s_t) n_{t-1}. \tag{1}$$

One typically thinks of cyclical fluctuations in (f_t, s_t) as being ultimately driven by more fundamental shocks governing the job creation policy of the firms and the natural breakdown of existing employment relationships. For example, endogenous variations in (f_t, s_t) naturally arise in a labour market plagued by search frictions, wherein both transition rates are affected by the underlying productivity shocks. We provide a model of such a frictional labour market in Appendix A.[10] However, we wish to emphasise here that the key market friction leading

[7] Krusell and Smith (1998) introduced heterogenous, stochastic discount factors to generate plausible levels of wealth dispersion in their incomplete-market environement. Our specification is closer to that in McKay and Reis (2012) who use heterogenous, but deterministic, discount factors.

[8] See, e.g., Becker (1980); Becker and Foias (1987); Kiyotaki and Moore (1997); Iacoviello (2005).

[9] This formulation is fully constitent with the possibility of unemployment spells shorter than a period (e.g., a quarter). Assume, for example, that an employed worker at the end of date $t-1$ looses its job at the beginning of date t with probability ρ_t, but is re-hired during the same period with probability f_t. Then, the period-to-period job-loss probability is $s_t = \rho_t (1-f_t)$, while the employment dynamics (1) still holds.

[10] See also Krusell et al. (2011), who solve numerically a full-fledged heterogenous-agent model with incomplete insurance and labour market frictions.

to time-varying precautionary savings is the inability of some households to perfectly insure against such transitions, a property that does not depend on the specific modelling of the labour market being adopted. For this reason, we take those transition rates as exogenous in our baseline specification, and will ultimately extract them from the data in the quantitative implementation of the model.

Impatient households. Impatient households maximise their expected life-time utility $E_0 \sum_{t=0}^{\infty} \left(\beta^I\right)^t u^I\left(c_t^i\right)$, $i \in [0, \Omega]$, where c_t^i is (nondurables) consumption by household i at date t and $u^I(.)$ a period utility function satisfying $u^{I\prime}(.) > 0$ and $u^{I\prime\prime}(.) \leq 0$. We restrict the set of assets that impatient households have access to in two ways. First, we assume that they cannot issue assets contingent on their employment status but only enjoy the (partial) insurance provided by the public unemployment insurance scheme; and second, we assume that these households face an (exogenous) borrowing limit in that their asset wealth cannot fall below $-\mu$, where $\mu \geq 0$.[11] Given these restrictions, the only asset that can be used to smooth out idiosyncratic labour income fluctuations are claims to the capital stock. We denote by e_t^i household's i employment status at date t, with $e_t^i = 1$ if the household is employed and zero otherwise. The budget and non-negativity constraints faced by an impatient household i are:

$$a_t^i + c_t^i = e_t^i w_t^I \left(1 - \tau_t\right) + \left(1 - e_t^i\right) \delta^I + R_t a_{t-1}^i, \tag{2}$$

$$c_t^i \geq 0,\ a_t^i \geq -\mu. \tag{3}$$

where a_t^i is household i's holdings of claims to the capital stock at the end of date t, R_t the ex post gross return on these claims, w_t^I the real wage for impatient households (assumed to be identical across them), δ^I the unemployment benefit enjoyed by these households when unemployed, and $w_t^I \tau_t$ a contribution paid by the employed and aimed at financing the unemployment insurance scheme.[12] The Euler condition for impatient households is:

$$u^{I\prime}\left(c_t^i\right) = \beta^I E_t \left(u^{I\prime}\left(c_{t+1}^i\right) R_{t+1}\right) + \varphi_t^i, \tag{4}$$

[11] In our model, this constraint will effectively binds only for the households who are both impatient and unemployed, a small fraction of the population (3.4% in our baseline calibration). The model can accomodate an endogenous borrowing limit based on limited commitment, e.g., if households pleageable income is a constant fraction $\tilde{\mu}$ of next period's expected future labour income. This does not significantly affect our results provided that pledgeable income is not too volatile. See Guerrieri and Lorenzoni (2011) for an analysis of how an exogenous change in the tightness of the constraint (as arguably occurred at the oneset of the Great Recession) affects outcomes under incomplete markets.

[12] Since households do not choose hours or participation here, it does not matter for aggregate dynamics whether social contributions are proportional or lump sum.

where φ_t^i is the Lagrange coefficient associated with the borrowing constraint $a_t^i \geq 0$, with $\varphi_t^i > 0$ if the constraint is binding and $\varphi_t^i = 0$ otherwise. Condition (4), together with the initial asset holdings a_{-1}^i and the terminal condition $\lim_{t\to\infty} E_t[\beta^{It} a_t^i u^{I\prime}(c_t^i)] = 0$, fully characterise the optimal asset holdings of impatient households.

Patient households. Patient households maximise $E_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t u^P(c_t^i)$, $i \in (\Omega, 1]$, where $\beta^P \in \left(\beta^I, 1\right)$ and $u^P(.)$ is increasing and strictly concave over $[0, \infty)$. In contrast to impatient households, patient households have complete access to asset markets –including the full set of Arrow-Debreu securities and loan contracts.[13] Full insurance implies that these households collectively behave like a large representative 'family' of permanent-income consumers in which the family head ensures equal marginal utility of wealth for all its members –despite the fact that individuals experience heterogenous employment histories (see, e.g., Merz, 1995; Hall, 2009). Since consumption is the only argument in the period utility, equal marginal utility of wealth implies equal consumption, so we may write the budget constraint of the family as follows:

$$C_t^P + A_t^P = R_t A_{t-1}^P + (1-\Omega)\left(n_t w_t^P (1-\tau_t) + (1-n_t)\,\delta^P\right), \tag{5}$$

where C_t^P (≥ 0) and A_t^P denote the consumption and end-of-period asset holdings of the family (both of which must be divided by $1-\Omega$ to find the per-family member analogues), and w_t^P and δ^P are the real wage and unemployment benefit for patient households, respectively. The Euler condition for patient households is given by:

$$u^{P\prime}\left(\frac{C_t^P}{1-\Omega}\right) = \beta^P E_t\left(u^{P\prime}\left(\frac{C_{t+1}^P}{1-\Omega}\right) R_{t+1}\right). \tag{6}$$

This condition, together with the initial asset holdings A_{-1}^P and the terminal condition $\lim_{t\to\infty} E_t[\left(\beta^P\right)^t A_t^P u^{P\prime}\left(C_t^P/(1-\Omega)\right)] = 0$, fully characterise the optimal consumption path of patient households. Note that when $\Omega = 0$ then only fully-insured patient households populate the economy, and the latter become a representative-agent economy.

[13] Patient households will be more wealthy than impatient households in equilibrium –and a lot more so when we calibrate the model to match the cross-sectional distribution of wealth in the US. Under fixed participation cost to trading Arrow-Debreu securities (as in, e.g., Mengus and Pancrasi (2012)), we expect households holding more wealth (patient households here) to be more willing to buy insurance, all else equal. From a quantitative point of view, Krusell and Smith (1998) have argued that the behaviour of *wealthy* agents facing incomplete markets and borrowing constraints is almost undistinguishable from that of fully insured agents. Finally, it is easy to show that in our economy the borrowing constraint would never be binding for fully-insured patient households in equilibrium, even if such a constraint were assumed in the first place (because patient households are relatively wealthy and claims to the capital stock are in positive net supply).

2.2 Production

The representative firm produces output, Y_t, out of capital, K_t, and the units of effective labour supplied by the households. Let n_t^I and n_t^P denote the firm's use of impatient and patient households' labour input, respectively, and $Y_t = z_t G\left(K_t, n_t^I + \kappa n_t^P\right)$ the aggregate production function, where $\kappa > 0$ is the relative efficiency of patient households' labour (with the efficiency of impatient households' labour normalised to one), $\{z_t\}_{t=0}^{\infty}$ a stochastic aggregate productivity process with unconditional mean $z^* = 1$, and where $G(.,.)$ exhibits positive, decreasing marginal products and constant returns to scale (CRS). As will be clear in Section 4 below, the introduction of an efficiency premium for patient households (i.e., $\kappa > 1$), which raises their labour income share in equilibrium relative to the symmetric case ($\kappa = 1$), is necessary to match the cross-sectional *consumption* dispersion, for any plausible level of wealth dispersion.[14] Defining $k_t \equiv K_t / \left(n_t^I + \kappa n_t^P\right)$ as capital per unit of effective labour and $g(k_t) \equiv G(k_t, 1)$ the corresponding intensive-form production function, we have $Y_t = z_t \left(n_t^I + \kappa n_t^P\right) g(k_t)$. Capital depreciates at the constant rate $\nu \in [0, 1]$, so that investment is $I_t \equiv K_{t+1} - (1-\nu) K_t$. Given R_t and z_t, the optimal demand for capital by the representative firm satisfies:

$$z_t g'(k_t) = R_t - 1 + \nu. \tag{7}$$

On the other hand, the optimal demands for the two labour types in a perfectly competitive labour market must satisfy $z_t G_2\left(K_t, n_t^I + \kappa n_t^P\right) = w_t^I = w_t^P/\kappa$, where w_t^I is the real wage per unit of effective labour.

2.3 Market clearing

By the law of large numbers and the fact that all households face identical transition rates in the labour market, the equilibrium numbers of impatient and patient households working in the representative firm are $n_t^I = \Omega n_t$ and $n_t^P = (1-\Omega) n_t$, respectively. Consequently, effective labour and capital are $n_t^I + \kappa n_t^P = (\Omega + (1-\Omega)\kappa) n_t$ (with n_t given by (1)) and

[14] Although we do not model it explicitly here, that $\kappa > 1$ is a direct implication of standard human capital accumulation models, which predict that more patient agents accumulate more human capital in the first place (e.g., Ben Porath, 1967). Our underlying assumption of perfect substituability between *efficient* labour units is made for simplicity, as it makes the equilibrium wage premium w_t^P/w_t^I constant and equal to the exogenous productivity premium κ. Introducing imperfect substituabilty between labour types (see, e.g., Acemoglu and Autor, 2011) makes the wage premium a function of the employment levels (n_t^P, n_t^I) but does not alter the basic mechanisms that we focus on.

$K_t = (\Omega + (1-\Omega)\kappa) n_t k_t$, respectively. Moreover, by the CRS assumption the equilibrium real wage per unit of effective labour is $w_t^I = z_t (g(k_t) - k_t g'(k_t))$.

In general, we expect incomplete insurance against unemployment shocks to generate cross-sectional wealth dispersion, as the asset wealth accumulated by a particular household depends on the employment history of this household. We summarise this heterogeneity in accumulated wealth by $F_t(\tilde{a}, e)$, which denotes the measure at date t of impatient households with beginning-of-period asset wealth $\tilde{a}$ and employment status e, and we denote by $a_t(\tilde{a}, e)$ and $c_t(\tilde{a}, e)$ the corresponding policy functions for assets and consumption.[15] Since those households are in share Ω in the economy, clearing of the market for claims to the capital stock requires that

$$A_{t-1}^P + \Omega \sum_{e=0,1} \int_{\tilde{a}=-\mu}^{+\infty} a_{t-1}(\tilde{a}, e)\, dF_{t-1}(\tilde{a}, e) = (\Omega + (1-\Omega)\kappa) n_t k_t, \tag{8}$$

where the left hand side of (8) is total asset holdings by all households at the end of date $t-1$ and the right hand side the demand for capital by the representative firm at date t. Clearing of the goods market requires:

$$C_t^P + \Omega \sum_{e=0,1} \int_{\tilde{a}=-\mu}^{+\infty} c_t(\tilde{a}, e)\, dF_t(\tilde{a}, e) + I_t = z_t (\Omega + (1-\Omega)\kappa) n_t g(k_t), \tag{9}$$

where the left hand side of (9) includes the consumption of all households as well as aggregate investment, $I_t = (\Omega + (1-\Omega)\kappa)(n_{t+1}k_{t+1} - (1-\nu) n_t k_t)$, and the right hand side is output. Finally, we require the unemployment insurance scheme to be balanced, i.e.,

$$\tau_t n_t \left(\Omega w_t^I + (1-\Omega) w_t^P\right) = (1-n_t)\left(\Omega \delta^I + (1-\Omega)\delta^P\right), \tag{10}$$

where the left and right hand sides of (10) are total unemployment contributions and benefits, respectively.

An *equilibrium* of this economy is defined as a sequence of households' decisions $\{C_t^P, c_t^i, A_t^P, a_t^i\}_{t=0}^{\infty}$, $i \in [0, \Omega]$, firm's capital per effective labour unit $\{k_t\}_{t=0}^{\infty}$, and aggregate variables $\{n_t, w_t^I, R_t, \tau_t\}_{t=0}^{\infty}$, which satisfy the households' and the representative firm's optimality conditions (4), (6) and (7), together with the market-clearing and balanced-budget conditions (8)–(10), given the forcing sequences $\{f_t, s_t, z_t\}_{t=0}^{\infty}$ and the initial wealth distribution $\left(A_{-1}^p, a_{-1}^i,\right)_{i\in[0,\Omega]}$.

[15] Our formulation of the market-clearing conditions (8)–(9) presumes the existence of a recursive formulation of the household's problem with $(\tilde{a}, e)$ as individual state variables, as this will be the case in the equilibrium that we are considering. See, e.g., Heathcote (2005) for a nonrecursive formulation of the household's problem.

3 An equilibrium with limited cross-sectional heterogeneity

As is well known, dynamic general equilibrium models with incomplete markets and borrowing constraints are not tractable, essentially because any household's decisions depend on its accumulated asset wealth, while the latter is determined by the entire history of idiosyncratic shocks that this household has faced. In consequence, the asymptotic cross-sectional distribution of wealth usually has an infinitely large number of states, and hence infinitely many agent types end up populating the economy (Aiyagari, 1994; Krusell and Smith, 1998). In this paper, we make specific assumptions about impatient household's period utility and the tightness of the borrowing constraint, which ensure that the cross-sectional distribution of wealth has a finite number of wealth states as an equilibrium outcome. As a result, the economy is characterised by a finite number of heterogenous agents whose behaviour can be aggregated exactly, thereby making it possible to represent the model's dynamics via a standard (small-scale) dynamic system. In the remainder of the paper, we focus on the simplest equilibrium, which involves exactly two possible wealth states for impatient households. However, we show in Section 3.3 and Appendix B how this approach can be generalised to construct equilibria with any finite number of wealth states.

3.1 Assumptions and conjectured equilibrium

Let us first assume that the instant utility function for impatient households $u^I(c)$ is i) continuous, increasing and differentiable over $[0, +\infty)$, ii) strictly concave with local relative risk aversion coefficient $\sigma^I(c) = -cu^{I\prime\prime}(c)/u^{I\prime}(c) > 0$ over $[0, c^*]$, where c^* is an exogenous, positive threshold, and iii) linear with slope $\eta > 0$ over $(c^*, +\infty)$ (see Figure 2). Essentially, this utility function (an extreme form of decreasing relative risk aversion) implies that high-consumption (i.e., relatively wealthy) impatient households do not mind moderate consumption fluctuations –i.e., as long as the implied optimal consumption level says inside $(c^*, +\infty)$– but dislike substantial consumption drops –those that would cause consumption to fall inside the $[0, c^*]$ interval.

Given this utility function, we derive our equilibrium with limited cross-sectional heterogeneity by construction; Namely, we first guess the general form of the solution, and then verify ex post that the set of conditions under which the conjectured equilibrium was derived does prevail in equilibrium. Our first conjecture is that an employed, impatient household is sufficiently wealthy for its chosen consumption level to lie above c^*, while an unemployed, impatient household chooses a consumption level below c^*. In other words, we are constructing an equilibrium in which the following condition holds:

$$\textbf{Condition 1}: \forall i \in [0, \Omega], \begin{cases} e_t^i = 1 \Rightarrow c_t^i > c^*, \\ e_t^i = 0 \Rightarrow c_t^i \leq c^*. \end{cases} \tag{11}$$

As we shall see shortly, one implication of this utility function and ranking of consumption levels is that employed households fear unemployment, and consequently engage in precautionary saving behaviour *ex ante* in order to limit (but without being able to fully eliminate) the associated rise in marginal utility.

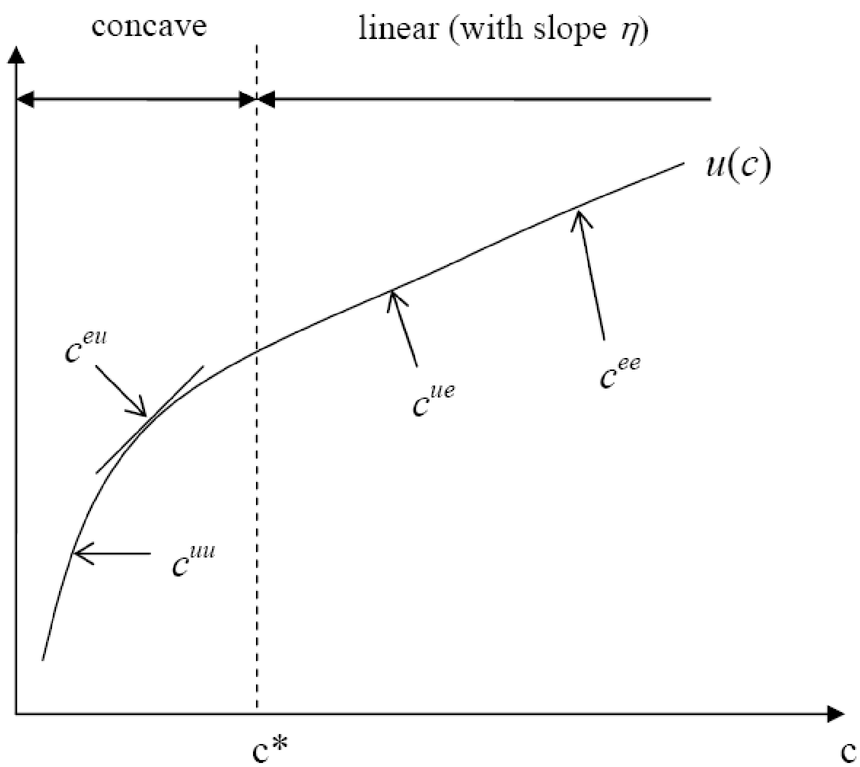

Figure 1. Instant utility function of impatient households, $u^I(c)$.

The second feature of the equilibrium that we are constructing is that the borrowing constraint in (3) is binding (that is, the Lagrange multiplier in (4) is positive) for all unemployed, impatient households, so that their end-of-period asset holdings are zero:

$$\textbf{Condition 2}: \forall i \in [0, \Omega], \ e_t^i = 0 \Rightarrow u'\left(c_t^i\right) > E_t\left(\beta^I u'\left(c_{t+1}^i\right) R_{t+1}\right) \text{ and } a_t^i = -\mu. \tag{12}$$

Equations (11)–(12) have direct implications for the optimal asset holdings of employed households. By construction, a household who is employed at date t has asset wealth $a_t^i R_{t+1}$

at the beginning of date $t+1$. If the household falls into unemployment at date $t+1$, then the borrowing constraint becomes binding and the household liquidates all assets. This implies that the household enjoys consumption

$$c_{t+1}^i = \delta^I + \mu + a_t^i R_{t+1} \tag{13}$$

and marginal utility $u^{I\prime}\left(\delta^I + \mu + a_{t+1}^i R_{t+1}\right)$.

There are now two cases to distinguish, depending on whether or not this household faces a binding borrowing constraint at date t (that it, when the household is still employed). If it does not, then $a_t^i > -\mu$ in (13), i.e., the household has formed a buffer of precautionary asset wealth in excess of the borrowing limit when still employed (with the buffer being of size $a_t^i + \mu > 0$). If it does, then $a_t^i = -\mu$ in (13) (so that $c_{t+1}^i = \delta^I - \mu\left(R_{t+1} - 1\right)$), and the household had consumed its entire (wage and asset) income at date t.

The precautionary saving case. If the borrowing constraint does not bind at date t, then $a_t^i > -\mu$ and the following Euler condition must hold at that date:

$$\eta = \beta^I E_t\left[\left(\left(1 - s_{t+1}\right)\eta + s_{t+1} u^{I\prime}\left(\delta^I + \mu + a_t^i R_{t+1}\right)\right) R_{t+1}\right]. \tag{14}$$

The left hand side of (14) is the current marginal utility of this household, which is equal to η under condition (11). The left hand side of (14) is expected, discounted future marginal utility, with marginal utility at date $t+1$ being broken into the two possible employment statuses that this household may experience at that date, weighted by their probabilities of occurrence (consequently, the expectations operator $E_t(.)$ in (14) is with respect to aggregate uncertainty only). More specifically, if the household stays employed, which occurs with probability $1-s_{t+1}$, it enjoys marginal utility η at date $t+1$ (by equation (11)); if the household falls into unemployment, which occurs with complementary probability, it liquidates assets (by equation (12)) and, as discussed above, enjoys marginal utility $u^{I\prime}\left(\delta^I + \mu + a_t^i R_{t+1}\right)$. Since equation (14) pins down a_t^i as a function of *aggregate* variables only (i.e., s_{t+1} and R_{t+1}), asset holdings are symmetric across employed households. Formally,

$$\forall i \in [0, \Omega], \; e_t^i = 1 \Rightarrow a_t^i = a_t. \tag{15}$$

To get further insight into how unemployment risk affects precautionary wealth, it may be useful to substitute (15) into (14) and rewrite the optimal asset holding equation for employed households as follows:

$$\beta^I E_t\left[\left(1 + s_{t+1}\frac{u^{I\prime}\left(\delta^I + \mu + a_t R_{t+1}\right) - \eta}{\eta}\right) R_{t+1}\right] = 1. \tag{16}$$

Consider, for the sake of the argument, the effect of a fully predictable increase in s_{t+1} holding R_{t+1} constant. The direct effect is to raise $1 + s_{t+1}[u^{I\prime}\left(\delta^I + \mu + a_t R_{t+1}\right) - \eta]/\eta$, since the proportional change in marginal utility associated with becoming unemployed, $[u^{I\prime}\left(\delta^I + \mu + a_t R_{t+1}\right) - \eta]/\eta$, is positive (see Figure 1). Hence, $u^{I\prime}\left(\delta^I + \mu + a_t R_{t+1}\right)$ must go down for (16) to hold, which is achieved by raising date t asset holdings, a_t. Later on we derive an approximate asset holding rule that explicitly connects current precautionary asset wealth to the expected job-loss rate and the expected interest rate.

The hand-to-mouth case. In the case where the borrowing constraint is binding for employed, impatient households –in addition to being binding for the unemployed, as conjectured in (12)–, then from (2) the consumption of employed households is $w_t(1-\tau_t) - \mu(R_t - 1)$ while that of unemployed households is $\delta^I - \mu(R_t - 1)$. In other words, all impatient households consume their entire (wage and asset) income in every period. Our model thus nests the pure "hand-to-mouth" behaviour as a special case, which occurs when the borrowing constraint is binding for all impatient households (and not only the unemployed). As we discuss below, this corner scenario notably arises when i) direct unemployment insurance is sufficiently generous (so that self-insurance is deterred), or impatient households' discount factor is sufficiently low (i.e., households are too impatient to save).

Aggregation. The analysis above implies that, under conditions (11)–(12), the cross-sectional distribution of wealth amongst impatient households at any point in time has at most two states: exactly two ($-\mu$ and $a_t > -\mu$) if the borrowing constraint is binding for unemployed households but not for employed households, and exactly one ($-\mu$) if the constraint is binding for all impatient households, employed and unemployed alike. This in turn implies that the economy is populated by at most *four* types of impatient households –since from (2) the type of a household depends on both beginning- and end-of-period asset wealth. We call these types 'ij', $i, j = e, u$, where i (j) refers to the household's employment status in the previous (current) date (for example, a 'ue household' is currently employed but was unemployed in the previous period, and its consumption at date t is c_t^{ue}). These consumption levels are :

$$c_t^{ee} = w_t(1-\tau_t) + R_t a_{t-1} - a_t, \tag{17}$$

$$c_t^{eu} = \delta^I + \mu + R_t a_{t-1}, \tag{18}$$

$$c_t^{ue} = w_t(1-\tau_t) - a_t - \mu R_t, \tag{19}$$

$$c_t^{uu} = \delta^I + \mu - \mu R_t. \tag{20}$$

where a_t is given by (16) in the precautionary-saving case and by $-\mu$ in the hand-to-mouth case (hence in the latter case $c_t^{ee}=c_t^{ue}$ and $c_t^{eu}=c_t^{uu}$). Finally, denoting by ω^{ij} the number of impatient households of type ij in the economy at date t, labour market flows imply that we have:

$$\omega_t^{ee}=\Omega\left(1-s_t\right)\left(\omega_{t-1}^{ee}+\omega_{t-1}^{ue}\right),\ \omega_t^{eu}=\Omega s_t\left(\omega_{t-1}^{ee}+\omega_{t-1}^{ue}\right), \tag{21}$$

$$\omega_t^{uu}=\Omega\left(1-f_t\right)\left(\omega_{t-1}^{eu}+\omega_{t-1}^{uu}\right),\ \omega_t^{ue}=\Omega f_t\left(\omega_{t-1}^{eu}+\omega_{t-1}^{uu}\right). \tag{22}$$

The limited cross-sectional heterogeneity that prevails across impatient households implies that we can exactly aggregate their asset holding choices. From (12) and (15), total asset holdings by impatient households is

$$\begin{aligned} A_t^I &\equiv \Omega\sum_{e=0,1}\int_{\tilde{a}=-\mu}^{+\infty} a_t\left(\tilde{a},e\right)dF_t\left(\tilde{a},e\right) \qquad (23)\\ &=\Omega\left(n_t a_t-\left(1-n_t\right)\mu\right), \end{aligned}$$

which can be substituted into the market-clearing condition (8). Similarly, aggregating individual consumption levels (17)–(20) given the distribution of types in (21)–(22), we find total consumption by impatient households to be:

$$\begin{aligned} C_t^I &\equiv \Omega\sum_{e=0,1}\int_{\tilde{a}=-\mu}^{+\infty} c_t\left(\tilde{a},e\right)dF_t\left(\tilde{a},e\right) \qquad (24)\\ &= \underbrace{\Omega\left(n_t w_t^I\left(1-\tau_t\right)+\left(1-n_t\right)\delta^I\right)+\left(R_t-1\right)A_{t-1}^I}_{\text{net income}}-\underbrace{\Omega\Delta\left(n_t\left(a_t+\mu\right)\right)}_{\text{change in asset wealth}}, \end{aligned}$$

where A_{t-1}^I is given by (23) and Δ is the difference operator (so that $\Delta A_t^I=\Omega\Delta\left(n_t\left(a_t+\mu\right)\right)$).

Equation (24) summarises the determinants of total consumption by impatient households in the economy. At date t, their aggregate net income is given by past asset accumulation and current factor payments –and hence taken as given by the households in the current period. The change in their total asset holdings, $\Omega\Delta\left(n_t\left(a_t+\mu\right)\right)$, depends on both the change in the number of precautionary savers Ωn_t (the "extensive" asset holding margin) and the assets held by each of them a_t (the "intensive" margin). The former is determined by employment flows is thus beyond the households' control, while the latter is their key choice variable. In the *precautionary saving* case, a_t is given by (16) and hence rises when labour market conditions are expected to worsen (i.e., s_{t+1} is expected to fall), which contributes to take C_t^I down. In the *hand-to-mouth* (*HTM*) case we simply have $a_t=-\mu$, so that $A_t^{I,HTM}=-\mu\Omega$ and

$$C_t^{I,HTM}=\Omega\left(n_t w_t^I\left(1-\tau_t\right)+\left(1-n_t\right)\delta^I-\mu\left(R_t-1\right)\right), \tag{25}$$

implying that only current labour market conditions affect $C_t^{I,HTM}$ (via their effect on n_t). Comparing (24) and (25), we get that:

$$C_t^I = C_t^{I,HTM} + \Omega \left(R_t n_{t-1} \left(a_{t-1} + \mu \right) - n_t \left(a_t + \mu \right) \right).$$

The latter expression shows how total consumption by impatient households –and, by way of consequence, aggregate consumption itself– differs across the hand-to-mouth and the precautionary-saving cases. In the hand-to-mouth case, only current labour market conditions n_t –in addition to factor prices (w_t^I, R_t)– affect total consumption by impatient households. In the precautionary-saving case, the same effects are at work but, in addition, future labour market conditions matter –inasmuch as they affect a_t. This suggests that the precautionary saving model may display *more* consumption volatility than the hand-to-mouth model, provided that labour market conditions are sufficiently persistent. This will be confirmed in the quantitative analysis of Section 4 below.

3.2 Existence conditions and steady state

Existence conditions. The equilibrium with limited cross-sectional heterogeneity described so far exists provided that two conditions are satisfied. First, the postulated ranking of consumption levels for impatient households in (11) must hold in equilibrium. Second, unemployed, impatient households must face a binding borrowing constraint (see (12)).

- From (17)–(20) and the fact that $a_t \geq -\mu$ (with equality in the hand-to-mouth case), we have $c_t^{uu} \leq c_t^{eu}$ and $c_t^{ee} \geq c_t^{ue}$. Hence, a necessary and sufficient condition for (11) to hold is $c_t^{eu} < c^* < c_t^{ue}$, that is,

$$\delta^I + \mu + a_{t-1} R_t < c^* < w_t^I \left(1 - \tau_t\right) - a_t - \mu R_t. \tag{26}$$

- Unemployed, impatient households can be of two types, uu and eu, and we require both types to face a binding borrowing constraint in equilibrium. However, since $c_t^{uu} \leq c_t^{eu}$ (and hence $u^{I\prime}\left(c_t^{uu}\right) \geq u^{I\prime}\left(c_t^{eu}\right)$), a necessary and sufficient condition for both types to be constrained is

$$u^{I\prime}\left(c_t^{eu}\right) > \beta^I E_t(\left(f_{t+1} u^{I\prime}\left(c_t^{ue}\right) + \left(1 - f_{t+1}\right) u^{I\prime}\left(c_{t+1}^{uu}\right)\right) R_{t+1}),$$

where the right hand side of the inequality is the expected, discounted marginal utility of an eu household who is contemplating the possibility of either remaining unemployed (with probability $1 - f_{t+1}$) or finding a job (with probability f_{t+1}). Under the conjectured equilibrium we have $u^{I\prime}\left(c_t^{ue}\right)=\eta$ and $c_{t+1}^{uu}=\delta^I+\mu\left(1-R_{t+1}\right)$, so the latter inequality becomes:

$$u^{I\prime}\left(\delta^I+\mu+a_{t-1}R_t\right)>\beta^I E_t\left(\left(f_{t+1}\eta+\left(1-f_{t+1}\right)u^{I\prime}\left(\delta^I+\mu\left(1-R_{t+1}\right)\right)\right)R_{t+1}\right). \quad (27)$$

In what follows, we compute the steady state of our conjectured equilibrium and derive a set of necessary and sufficient conditions for (26)–(27) to hold in the absence of aggregate shocks. By continuity, they will also hold in the stochastic equilibrium provided that the magnitude of aggregate shocks is not too large.

Steady state. In the steady state, the real interest rate is determined by the discount rate of the most patient households, so that $R^*=1/\beta^P$ (see (6)). From (1) and (7), the steady state levels of employment and capital per effective labour unit are

$$n^*=\frac{f^*}{f^*+s^*},\ k^*=g'^{-1}\left(\frac{1}{\beta^P}-1+\nu\right). \quad (28)$$

A key variable in the model is the level of asset holdings that employed, impatient households hold as a buffer against unemployment risk. If the borrowing constraint is binding in the steady state, then they never hold wealth. The interior solution to the steady state counterpart of (16) (where $R^*=1/\beta^P$) gives the individual asset holdings:

$$\tilde{a}^*=\beta^P\left[u^{I\prime-1}\left(\eta\left(1+\frac{\beta^P-\beta^I}{\beta^I s^*}\right)\right)-\delta^I-\mu\right]. \quad (29)$$

The borrowing constraint is binding whenever the interior solution $\tilde{a}^*$ in (29) is less than $-\mu$. Hence the actual steady-state wealth level of employed, impatient households is given by:

$$a^*=\max\left[-\mu,\tilde{a}^*\right], \quad (30)$$

which nests both the precautionary-saving and hand-to-mouth cases discussed above.

Equations (29)–(30) are informative about the conditions under which the economy collapses to a hand-to-mouth economy. More specifically, employed, impatient households form no buffer stock of wealth whenever $\tilde{a}^*<-\mu$, that is, using (29)–(30) and rearranging, whenever

$$\frac{\beta^P-\beta^I}{\beta^I}>s^*\times\frac{u'\left(\delta^I+\mu-\mu/\beta^P\right)-\eta}{\eta}. \quad (31)$$

This inequality is straightforward to interpret. In the hand-to-mouth case, the steady state consumption level of an impatient household who is transiting into unemployment is $\delta^I+\mu-\mu/\beta^P$ –that is, the unemployment benefit δ^I plus new borrowing μ minus the debt repayment

μ at the gross interest rate $1/\beta^P$. Thus, the right hand side of (31) is the proportional cost –in terms of marginal utility– associated with a completely unbuffered transition from employment (where marginal utility is η) to unemployment (where marginal utility is $u'\left(\delta^I + \mu - \mu/\beta^P\right)$), weighted by the probability of this transition occuring (the job-loss probability s^*). The higher the probability of this transition, the stronger the incentive to buffer the shock and the less likely (31) will hold, all else equal. Conversely, the higher the unemployment benefit δ^I, the lower the actual cost of the transition when it occurs, the weaker the incentive to hold a buffer stock, and the more likely (31) will hold; formally, $a^*\left(\delta^I\right)$ is a nonincreasing, continuous piecewise linear function with a kink at the value of δ^I for which $\tilde{a}^* = -\mu$ in (29) –see Figure 2. Finally, the left hand side of (31) measures the relative impatience of impatient households; the more impatient they are, the less willing to save and the more likely (31) will hold.

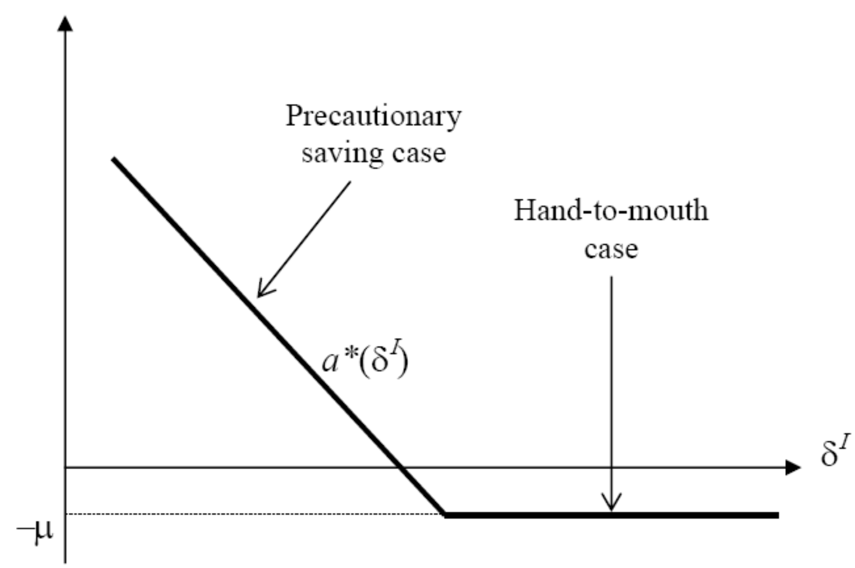

Figure 2. Unemployment insurance and precautionary saving.

Finally, from (8) and (23), steady state (total) asset holdings by impatient and patient households are $A^{I*} = \Omega\left(n^* a^* - (1 - n^*)\mu\right)$ and $A^{P*} = \left(\Omega + (1 - \Omega)\kappa\right) n^* k^* - A^{I*}$, respectively, where n^*, k^* and a^* are given by (28) and (30). The wealth share of the poorest $\Omega\%$ –one of our calibration targets below– is thus:

$$\frac{A^{I*}}{K^*} = \frac{\Omega\left(n^* a^* - (1 - n^*)\mu\right)}{\left(\Omega + (1 - \Omega)\kappa\right) n^* k^*} \tag{32}$$

The other relevant steady state values directly follow, notably the total consumption levels of patient and impatient households, and the consumption share of the bottom $\Omega\%$ –another calibration target–, $C^{I*}/\left(C^{I*} + C^{P*}\right)$.

We may now state the following proposition, which establishes the conditions on the deep parameters of the model under which a steady state with limited cross-sectional heterogeneity exists. Provided that aggregate shocks have sufficiently small magnitude, the same conditions will ensure the existence of a stochastic equilibrium with similarly limited heterogeneity.

Proposition 1. *Assume that i) there are no aggregate shocks, ii) unemployment insurance is incomplete (i.e., $\delta^I < w^{I*}(1-\tau^*)$) and iii) the following inequality holds:*

$$\eta\left(1+\frac{\beta^P-\beta^I}{\beta^I s^*}\right) >$$
$$\max\left[\frac{\beta^I}{\beta^P}\left(f\eta+(1-f)\,u'\left(\delta^I-\mu\left(\frac{1}{\beta^P}-1\right)\right)\right),u'\left(\frac{w^{I*}(1-\tau^*)+\beta^P\delta^I}{1+\beta^P}-\mu\left(\frac{1}{\beta^P}-1\right)\right)\right],$$

where

$$\tau^*=\left(\frac{1-n^*}{n^*}\right)\frac{\Omega\delta^I+(1-\Omega)\,\delta^P}{(\Omega+(1-\Omega)\,\kappa)\,w^{I*}}, \tag{33}$$

$w^{I*}=g(k^*)-k^*g'(k^*)$, *and* (n^*,k^*) *are given by (28). Then, it is always possible to find a utility threshold* c^* *such that the conjectured limited-heterogeneity equilibrium described above exists. In this equilibrium,* $a^*=-\mu$ ($a^*>-\mu$) *if (31) holds (does not hold).*

Proof. First, the steady state counterpart of (27) is

$$a^*<\beta^P u^{I\prime-1}\left[\frac{\beta^I}{\beta^P}\left(f\eta+(1-f)\,u'\left(\delta^I+\mu\left(\frac{\beta^P-1}{\beta^P}\right)\right)\right)\right]-\beta^P\left(\delta^I+\mu\right). \tag{34}$$

Second, the steady state counterpart of (26) is $\delta^I+\mu+a^*/\beta^P<c^*<w^{I*}(1-\tau_t)-a^*-\mu/\beta^P$. A sufficient condition for the existence of a threshold c^* is thus that $\delta^I+\mu+a^*/\beta^P< w^{I*}(1-\tau_t)-a^*-\mu/\beta^P$, or

$$a^*<\frac{\beta^P\Gamma}{1+\beta^P}-\mu, \tag{35}$$

where $\Gamma\equiv w^{I*}(1-\tau^*)-\delta^I=(1-\tau^*)\left[g(k^*)-k^*g'(k^*)\right]-\delta^I$ is a strictly positive constant that only depends on the deep parameters of the model (see (28) and (33)). Inequalities (34)–(35) hold for $a^*=-\mu$ (the hand-to-mouth case). Otherwise, a^* is given by (29) (the precautionary saving case); substituting this value of a^* into (34)–(35) and rearranging gives the inequality in the proposition■

The inequalities in Proposition 1 ensure that, in the steady state, i. the candidate equilibrium features at most two possible wealth levels for impatient households ($-\mu$ for the unemployed and $a^*\geq-\mu$ for the employed); and ii. the implied ranking of individual consumption levels is indeed such that we can "reverse-engineer" an instant utility function for these households of the form depicted in Figure 1. Those inequalities can straightforwardly be checked once specific values are assigned to the deep parameters of the model. As we argue in Section 4 below, it is satisfied for plausible such values when we calibrate the model to the US economy. The reason for which it does is as follows. Our limited-heterogeneity equilibrium requires that impatient, unemployed households be borrowing-constrained (i.e., they would like to borrow against future income but are prevented from doing so), while impatient, employed households accumulate sufficiently little wealth in equilibrium (so that this wealth be exhausted within a quarter of unemployment). In the US, the quarter-to-quarter probability of leaving unemployment is high and the replacement ratio relatively low, leading the unemployed's expected income to be sufficiently larger than current income for these households to be willing to borrow. On the other hand, the US distribution of wealth is fairly unequal, leading a large fraction of the population (the impatient in our model) to hold a very small fraction of total wealth.

Linearised asset holding rule. We conclude this section by stressing that, in case employed, impatient households do form precautionary saving, local time-variations in the probability to become unemployed, s_{t+1}, have a *first-order* effect on precautionary asset accumulation at the individual level, a_t. This is because even without aggregate risk a change in employment status from employment to unemployment at date $t+1$ is associated with a discontinuous drop in individual consumption, and hence with a infra-marginal rise in marginal utility from η to $u^{I\prime}(c^{eu}) > \eta$.[16] The probability s_{t+1} weights this possibility in employed households' Euler equation (see (16)), so even small changes in s_{t+1} have a sizeable impact on asset holdings and consumption choices. Linearising (16) around the steady state calculated above, we find the following approximate individual asset holding rule:

$$a_t \simeq a^* + \Gamma_s E_t \left(s_{t+1} - s^*\right) + \Gamma_R E_t \left(R_{t+1} - R^*\right),$$

with

$$\Gamma_s = \frac{\left(\beta^P - \beta^I\right)\left(\beta^P\left(\delta^I + \mu\right) + a^*\right)}{\left(\beta^P - \beta^I\left(1 - s^*\right)\right) s^* \sigma^I \left(c^{eu*}\right)} > 0,$$

[16] This property distinguishes our model from those which root the precautionary motive into households' 'prudence' (Kimball, 1990). In that framework, time-variations in precautionary savings may follow from changes in the second-order term of future marginal utility (see, e.g., Gourinchas and Parker, 2001; Parker and Preston, 2005). It is apparent from (16) that a mean-preserving increase in employed households' uncertainty about future labour income taking the form of an increase in s_{t+1} (and a corresponding rise in w_{t+1} to keep expected income constant) raises asset holdings –the usual definition of 'precautionary saving.'

and where a^* is given by (29), $c^{eu*} = \delta^I + \mu + a^* R^*$ is the steady state counterpart of (18), and $\sigma^I (c^{eu*}) \equiv -c^{eu*} u^{I''} (c^{eu*}) / u^{I'} (c^{eu*})$ is the coefficient of relative risk aversion of impatient households evaluated at c^{eu*} (i.e., the steady state consumption level of a household falling into unemployment). The composite parameter Γ_s measures the strength of the response of individuals' precautionary wealth following predicted changes in unemployment risk, such as summarised by the period-to-period separation rate s_{t+1}.[17]

3.3 Equilibria with multiple wealth states

In the previous sections, we have constructed an equilibrium with limited cross-sectional heterogeneity characterised by the simplest (nondegenerate) distribution of wealth, that with two states. A key feature of this equilibrium is that impatient households face a binding borrowing constraint after the first period of unemployment –and hence liquidate their entire asset wealth. As we argue next, instant asset liquidation by wealth-poor households is a natural outcome of our framework when we calibrate it on a quarterly basis and using US data on the cross-sectional distribution of wealth. However, we emphasise that the same approach can be use to construct equilibria with any finite number of wealth states, wherein households gradually, rather than instantly, sell assets to offset their individual income fall.

We carefully derive in Appendix B a set of necessary and sufficient conditions for the existence and uniqueness of limited-heterogeneity equilibria with $m + 1$ wealth states (that is, m strictly positive wealth states), thereby generalising the constructive approach used in Sections 3.1–3.2. As before, we focus on those conditions at the steady state, and resort to perturbation arguments to extend them to the stochastic equilibrium. An equilibrium with $m + 1$ wealth states has the property that the only impatient households facing a binding borrowing constraint are those having experienced at least m consecutive periods of unemployment. Before the mth unemployment period, the asset wealth and consumption level of these households decreases gradually. From the $m + 1$th unemployment period, these households face a binding borrowing constraint, hold no wealth, and have a flat consumption path. Finally, we show that an equilibrium with $m + 1$ wealth states is associated with $2(m + 1)$ types of impatient households (that is, the cross-sectional distribution of consumption has $2(m + 1)$ possible states).

To see this intuitively how this gradual process of asset decumulation can occur in our

[17] Γ_R may be positive or negative depending on the relative strengths of the intertemporal income and substitution effects. In particular, high values of σ^I produce asset accumulation rules that prescribe an increase in a_t following a fall in $E_t (R_{t+1})$.

framework, consider for simplicity the case where $\mu = 0$ and consider the steady state of the simple equilibrium described above and take its existence condition with respect to the bindingness of the borrowing constraint for an impatient households who fall into unemployment:

$$u^{I\prime}\left(\delta^I + a^* R^*\right) > \beta^I \left[\left(\eta f^* + (1 - f^*) \, u^{I\prime}\left(\delta^I\right)\right) R^*\right], \tag{36}$$

where $\delta^I + a^* R^*$ is the consumption of those households under full liquidation, η their marginal utility in the next period if they exit unemployment, and δ^I their consumption in the next period if they stay unemployed (with no assets left, by construction).

The circumstances leading to the violation of inequality (36), so that the equilibrium with two wealth states described above no longer exists, are the following. First, the job-finding rate f^* or the unemployment benefit δ^I may be too low, leading to high marginal utility in the next period (the right hand side of (36)), thereby urging the household to transfer wealth into the future. Second, the asset holdings accumulated when employed (a^*, which is itself determined by (29)) may be too high, leading to low current marginal utility (the left hand side (36)), thereby making this transfer little costly to the household. However, even if inequality (36) is violated for one of these reasons, a similar condition might nevertheless hold for households having experienced *two* consecutive periods of unemployment, because those have less wealth (and hence higher current marginal utility) than in the first unemployment period. In this case, the equilibrium will have exactly three wealth states (including two strictly positive), rather than two.

4 Time-varying precautionary saving and consumption fluctuations

The model developed above implies that some households rationally respond to countercyclical changes in unemployment risk by raising precautionary wealth –and thus by cutting down individual consumption more than they would have done without the precautionary motive. We now wish to assess the extent of this effect on aggregate consumption when realistic unemployment shocks are fed into our model economy. To this purpose, we compute the response of aggregate consumption and output to aggregate shocks implied by our baseline model, and then compare it with the data as well as a number of comparable benchmarks including i. the hand-to-mouth model, ii. the representative-agent economy, and iii. Krusell & Smith (1998) "stochastic-beta" economy.

4.1 Summary of the baseline precautionary-saving model

We start by writing down the dynamic system summarising the behaviour of our baseline precautionary-saving model, at the level of aggregation that is relevant for the quantitative exercises that follow. The model includes three forcing variables (z_t, f_t and s_t) and nine endogenous variables, namely: employment and capital per effective labour unit, n_t and k_t,; the total consumption of impatient and patient households, C_t^I and C_t^P; the corresponding asset levels, i.e., A_t^P for the representative family and a_t for an employed, impatient household; the factor prices R_t and w_t^I; and the unemployment contribution rate, τ_t. These endogenous variables are linked through the following equations:

$$\beta^I E_t \left[\left(1 + s_{t+1} \left(\frac{u^{I\prime} \left(\delta^I + \mu + a_t R_{t+1} \right) - \eta}{\eta} \right) \right) R_{t+1} \right] = 1, \tag{EE-I}$$

$$C_t^I + A_t^I = \Omega \left(n_t w_t^I \left(1 - \tau_t \right) + \left(1 - n_t \right) \delta^I \right) + R_t A_{t-1}^I, \tag{BC-I}$$

$$A_t^I = \Omega \left(n_t a_t - \left(1 - n_t \right) \mu \right), \tag{A-I}$$

$$\beta^P E_t \left(\frac{u^{P\prime} \left(C_{t+1}^P / \left(1 - \Omega \right) \right)}{u^{P\prime} \left(C_t^P / \left(1 - \Omega \right) \right)} R_{t+1} \right) = 1, \tag{EE-P}$$

$$C_t^P + A_t^P = \left(1 - \Omega \right) \left(\kappa n_t w_t^I \left(1 - \tau_t \right) + \left(1 - n_t \right) \delta^P \right) + R_t A_{t-1}^P, \tag{BC-P}$$

$$R_t = z_t g' \left(k_t \right) + 1 - \mu, \tag{IR}$$

$$w_t^I = z_t \left(g \left(k_t \right) - k_t g' \left(k_t \right) \right), \tag{WA}$$

$$A_{t-1}^P + A_{t-1}^I = \left(\Omega + \left(1 - \Omega \right) \kappa \right) n_t k_t, \tag{CM}$$

$$\tau_t n_t w_t^I \left(\Omega + \left(1 - \Omega \right) \kappa \right) = \left(1 - n_t \right) \left(\Omega \delta^I + \left(1 - \Omega \right) \delta^P \right), \tag{UI}$$

$$n_t = \left(1 - n_{t-1} \right) f_t + \left(1 - s_t \right) n_{t-1}. \tag{EM}$$

Equations (EE-I)–(A-I) are the Euler condition and aggregate budget constraint for impatient households –where (BC-I)–(A-I) are just (23)–(24). Equations (EE-P)–(BC-P) are the same conditions for patient households, such as given by (5)–(6) and where w_t^P has been replaced by its equilibrium value, κw_t^I. (IR) follows from the representative firm's optimality condition (7), with the factor price frontier under CRS giving w_t^I in (WA). Equation (CM) is the market-clearing condition for capital, which follows from substituting (23) into (8). Finally, (UI) is the balanced-budget condition for the unemployment insurance scheme (where again $w_t^P = \kappa w_t^I$ has been substituted into (10)), while (EM) is the law of motion for employment.

4.2 Alternative benchmarks

In what follows, we shall compare the time-series properties of our baseline precautionary saving model with the following alternative benchmarks.

Hand-to-mouth model. As discussed in Section 3, the hand-to-mouth model arises endogenously as a particular case of our framework whenever (31) holds (and aggregate shocks have sufficiently small magnitude). In this equilibrium all impatient households face a binding borrowing constraint in every period, so that $a_t = -\mu$ for all t. The dynamics of the hand-to-mouth model is obtained by removing equation (EE-I) from the dynamic system (EE-I)-(EM) above and by imposing $a_t = -\mu$ in equation (A-I).

Representative-agent model. Our framework also nests the representative-agent model as a special case. The comparable representative-economy is obtained by setting $\Omega^{RA} = 0$ –so that all households are identical and perfectly insured– and $\kappa^{RA} = \Omega + (1 - \Omega)\,\kappa$ –so that average labour productivity is kept the same as in the baseline precautionary-saving model. The subjective discount factor β^P is kept at the same value as in the baseline model, so that the steady state interest factor $R^* = 1/\beta^P$, and hence capital per effective labour unit k^*, are unchanged. Since steady-state total wealth is $(\Omega + (1 - \Omega)\,\kappa)\,n^* k^*$ (see (8)), it takes the same value in the representative-agent model as in the baseline precautionary-saving model (and also the hand-to-mouth model).

Krusell-Smith model. We also compare the quantitative implications of our model with the stochastic-beta version of the Krusell and Smith (1998) heterogenous-agent model. We thus simulate exactly the same model and then compute the moments that we wish to compare to those implied by our baseline model with limited cross-sectional heterogeneity.[18] Our motivation for focusing on the stochastic-beta version of the model is twofold. First, it incorporates discount factor heterogeneity which, as our model, potentially generates a substantial amount of cross-sectional wealth dispertion (as is usually observed empirically, notably in the US). Second, the stochastic-beta model is the Krusell-Smith variant that differs most from the full-insurance model in term of the consumption-output correlation, one of the key moments we are also interested in (see Table 2 in Krusell and Smith, 1998).

[18] We refer the reader to their paper for the description of their model and results (and notably Section 4 for the stochastic-beta model).

4.3 Calibration

We set the time period to a quarter (and thus allow unemployment spells to be shorter than a period –see footnote 7), and calibrate the model so that its steady state matches some key features of the cross-sectional distributions of wealth and consumption in the US economy (see Table 1 below). We then simulate the model to infer its its time-series properties under this calibration as well as alternative specification (as summarised in Table 2).

Idiosyncratic risk and insurance The extent of idiosyncratic risk that households face is measured by the labour market transition rates (f^*, s^* at the steady state), while the degree of insurance that they enjoy depends on both the replacement ratio δ^I/w^{*I} and the borrowing limit μ. The steady state values of f^* and s^* are set to their quarter-to-quarter post-war averages (see Appendix C for how quarterly series for f_t and s_t are constructed). By construction, these values produce a steady-state unemployment rate $s^*/(f^* + s^*)$ also equal to its post-war average, 5.65%.

In a narrow sense, the gross replacement ratio δ^j/w^{*j}, $j = I, P$, measures the replacement income provided by the unemployment insurance scheme and should thus be set between 0.4 and 0.5 for the US (see, e.g., Shimer, 2005; Chetty, 2008). However, households also benefit from other, nonobservable sources of insurance –family, friends etc.– We take this into account by calibrating δ^j/w^{*j} so as to generate a plausible level of consumption insurance. Cochrane (1991) argues that the average nondurable consumption growth of consumers experiencing an involuntary job loss is 25 percentage point lower than those who do not. Gruber (1997) focuses on the impact of UI benefits on the size of the consumption fall of households having experienced a job loss, and find a significantly smaller number –about 7 percent. We set our baseline gross replacement ratio δ^j/w^{*j} to 0.6 (rather than, say, 0.5) which, together with the other parameters of the model, produces a consumption growth differential of 14.08% for the average households;[19] we also evaluate the impact of a larger replacement ratio in our sensitivity experiments.

In the model, only the households who are both impatient –and hence have little assets in the first place– *and* unemployed face a binding borrowing constraint. In our baseline scenario,

[19] Patient households are fully insured and hence experience noconsumption fall when falling into unemployemnt. Hence, the average proportional consumption drop associated with falling into unemployment is $\Omega\left(c^{e*} - c^{eu*}\right)/c^{e*}$, where $c^{e*} \equiv f^*\left(1 - n^*\right)c^{ue*} + (1 - s)n^* c^{ee*}$ is the average consumption of and employed, impatient households. Our calibrated replacement ratio being perfectly symmetric across households, we are implicitly ignoring the potential redistributive effects of the unemployemnt insurance scheme.

we simply assume that these househoulds cannot borrow (i.e., $\mu = 0$), and then evaluate the impact of a relaxation of this constraint in our sensitivity analysis.[20] As we shall see, relaxing the borrowing within a plausible range significantly affects the cross-sectional distribution of wealth but not the time-series implications of the model.

Preferences. On the households' side, we adopt the following baseline parameters. We set the share of impatient households, Ω, to 0.6 (and experiment with an alternative value of 0.3 in our sensitivity analysis); since only a fraction $1 - n^*$ of such households face a binding borrowing constraint in the baseline precautionary-saving model, and given the calibrated steady-state transition rates f^* and s^* (see above), this implies a steady-state share of effectively borrowing-constrained households of $\Omega (1 - n^*) = 3.4\%$.[21] The discount factor of patient household, β^P $(= 1/R^*)$, is set to 0.99, and their instant utility to $u^P(c) = \ln c$.

The utility function of impatient households is set to

$$u^I(c) = \begin{cases} \ln c & \text{for } c \leq 1.6 \\ \ln 1.6 + 0.486 (c - 1.6) & \text{for } c > 1.6, \end{cases}$$

which satisfies the assumptions in Section 2.1 (with $\eta = 0.486$ and $c^* = 1.6$). The chosen value of η is equal to the steady state marginal utility of ee households (by far the most numerous amongst the impatient) if they had the same instant utility function as patient households – given the other parameters–, and is meant to minimise differences in asset holding behaviour purely due to differences in instant utility functions.[22] Note that $u^I(c)$ is continuous and (weakly) concave but not differentiable all over $[0, \infty)$ (since $u^{I\prime}(1.6) > 0.486$); however, it can be made so by 'smooth pasting' the two portions of the function in an arbitrarily small neighbourhood of c^* while preserving concavity. We set the discount factor of impatient households, β^I, to match the wealth share of the $\Omega\%$ poorest households, given the other parameters including β^P. We focus on nonhome (or "liquid") wealth, since our analysis

[20] Sullivan (2008) finds the response of (unsecured) individual debt to unemployment shocks to be inexistent or very small (a tenth of the associated income loss) in the US.

[21] Model with "rule-of-thumb" consumers usually have a larger fraction of effectively constrained households, ranging from 15% to 60% (see, e.g., Campbell and Mankiw, 1989; Iacoviello, 2005; Gali et al., 2007; Mertens and Ravn, 2011; Kaplan and Violante, 2012). In the hand-to-mouth economy, the share of effectively constrained household is raised from $(1 - n^*)\,\Omega$ $(= 3.4\%)$ to Ω $(= 60\%)$ (since all impatient households, including the employed, are then constrained).

[22] More specifically, η solves $\eta = u^{P\prime}(c^{ee*}) = \left(w^{I*} + a^* \left(1 - 1/\beta^P\right)\right)^{-1}$. This equation indicates that the appropriate value of η depends on a^*. Since a^* also depends on η (by (29)), we jointly solve for the fixed point (a^*, η) using a iterative procedure.

pertains to the the part of households' net worth that can readily be converted into cash to provide for current (nondurables) consumption. A value of $\beta^I = 0.9697$ produces a wealth share of 0.30% for the 60% poorest households (the A^{I*}/K^* ratio in (32)), which matches the corresponding quantile of the distribution of nonhome wealth in the 2007 Survey of Consumer Finances (see Wolff, 2011, Table 2).[23]

Technology. On the production side, we assume a Cobb-Douglas production function $Y_t = z_t K_t^\alpha \left(n_t^I + \kappa n_t^P\right)^{1-\alpha}$, with $\alpha = 1/3$, and a depreciation rate $\nu = 2.5\%$. The productivity premium parameter κ on patient households' labour is set to 1.731. Since impatient households' productivity is normalised to 1, this implies an equilibrium skill premium of 73.1%). Given the other parameters (see below), this will produce a consumption share $C^{I*}/(C^{I*} + C^{P*})$ of 40.62% for the 60% poorest households, which exactly matches the cross-sectional distribution of nondurables in the 2009 Consumer Expenditure Survey.[24] Note that this value is also well in line with the direct evidence on the level of the skill premium such as reported in, e.g., Heathcote et al. (2010) or Acemoglu and Autor (2011).

Our baseline parameterisation is summarised in Table 1. Given those parameters, and in particular the implied low wealth share of impatient households, the existence conditions summarised in Proposition 1 are satisfied (by a large margin). In particular, households who fall into unemployment without enjoying full insurance exhaust their buffer stock of wealth within a quarter, and thus live entirely out of unemployment benefits thereafter. Given the baseline values of Ω and κ, the representative-agent economy (i.e., in which $\Omega^{RA} = 0$) must be parametrised with a skill premium parameter $\kappa^{RA} = \Omega + (1 - \Omega)\,\kappa = 0.6 + 0.4 \times 1.731 = 1.292$. Finally, from (31) the calibrated model becomes a hand-to-mouth model whenever the steady state gross replacement ratio for impatient households, δ^I/w^{I*}, exceeds 0.694.

[23] According to Gruber (1998), the mean probability of selling one's home for those who become unemployed is only 3% –that is, households typically do not sell their home to smooth nondurables consumption. Besides the net ownership of the primary residence, the nonhome wealth distribution reported in Wolff (2011) excludes consumer durables (whose resale value is low) as well as the social security and pension components of wealth (which cannot be marketed). See Wolff (2011) for a full discussion of this point.

[24] Our empirical couterpart for the consumption share of impatient households is the share of nondurables consumed by the bottom three quintiles of households in terms of pre-tax income, where we define nondurables as in Heathcote et al. (2010).

Parameters	Symb.	Value	*Steady state* (%)	Value	Data	Source
Share of impatient hous.	Ω	.6	Unemployment rate	5.65	5.65	CPS
Disc. factor (patient)	β^P	.99	Liquid wealth share of			
Disc. factor (impatient)	β^I	.9697	bottom Ω%	0.30	0.30	SCF
Replacement ratio	δ/w	.6	Consumption share of			
Borrowing limit	μ	.0	bottom Ω%	40.62	40.62	CEX
Skill premium parameter	κ	1.731	Mean cons. fall after			
Capital share	α	1/3	unemployment shock	14.08	$[7, 25]$	See text
Depreciation rate	ν	.025				
Job-finding rate	f^*	.789				
Job separation rate	s^*	.047				

Table 1. Baseline model: parameters and implied steady state.

Note: The model matches the mean unemployment rate by construction. $\beta^I$and κ are set to that the wealth and consumption shares of the model (column 5) match their empirical counterparts (column 6), given the other parameters of the model (see text for details).

4.4 Aggregate consumption volatility

Experiment. We compute the second-order moment properties of the various specifications under consideration, focusing on the implied volatility of aggregate consumption and its correlation with output (see Table 2). For the three variants of system (EE-I)–(EM) under consideration (that is, the precautionary-saving model, the hand-to-mouth model and the represetative-agent model), we proceed as follows. We first estimate the joint behaviour of the exogenous state vector over the entire postwar period, using a four-lag VAR $X_t = \sum_{j=1}^{4} A_j X_{t-j} + \varepsilon_t$, where $X_t = \left[\tilde{f}_t, \tilde{s}_t, \tilde{z}_t\right]'$ includes the HP-filtered job-finding rate, job-separation rate and log-TFP, and where ε_t is the 1×3 vector of residuals; this gives us A_j, $j = 1...4$ as well as the covariance matrix $\Sigma \equiv \text{Var}(\varepsilon_t)$ (see Appendix C for details.). We then log-linearise the three model variants and solve for their VAR representation $Z_t = BZ_{t-1} + X_t$, where Z_t is the relevant vector of endogenous variables.[25] Last, we run stochastic simulations of each models with repeated shocks on X_t that have the same stochastic properties (in terms of autocorrelation and innovations' covariance structure) as those estimated from the data. In each case, the standard deviation of consumption Std(C) –the average proportional deviation

[25] Our results are almost identical when we consider second-order approximations of each model under consideration.

from trend, in %– is computed, as well as its correlation with output, $\text{Corr}(Y_t, C_t)$.

As discussed above, we also compare these moments with those implied by the stochastic-beta, Krusell and Smith (1998) model.[26] Because that model is not tractable, it has a single, two-state exogenous state variable which affects both total factor productivity and labour-market transition rates. This is in contrast with the other three stochastic experiments described above, which use a somewhat richer structure of aggregate shocks (namely, the estimated joint behaviour of transition rates and total factor productivity). Since we are simulating the same model, our value of $\text{Corr}(Y_t, C_t)$ in Table 2 below exactly replicates theirs (see Krusell and Smith, 1998, Table 2, p. 886). We also compute the wealth share of the 60% poorest as well as the standard deviation of aggregate consumption; in the later calculation, the simulated consumption series is loged and HP-filtered (with smoothing parameter 1600) before computing the standard deviation, so that the latter is comparable with those generated by the other models (which are expressed in percent deviation from trend).

Results. The top part of Table 2 reports, for each model under consideration (Model 2 to 5), the wealth share of the poorest Ω% (in the baseline scenario where $\Omega = 0.6$) as well as the second-order moments under investigation, and compare them with the data (Row 1). The bottom part of Table 2 carries out a number of sensitivity experiments around the baseline precautionary-saving model

The two models that are closest to the data are the baseline precautionary-saving model (Model 2) and the Krusell-Smith model (Model 4). As argued above, Model 2 has been parameterised to match the empirical share of liquid wealth held by the poorest 60% –hence the identity between the two in the Table (=0.30); in contrast, Model 4 was parameterised by Krusell and Smith (1998, Section IV) to fit the Lorenz curve for net worth; since net worth is less unequally distributed than liquid wealth, it generates a greater corresponding wealth share than does Model 2. As is extensively discussed in Krusell and Smith (1998), the wealth distribution in Model 4 implies that a large fraction of the households are reasonably well self-insured against idiosyncratic shocks, implying that their behaviour is close to that of pure permanent-income consumers; this shows up here as a smaller consumption volatility generated by Model 4 relative to Model 2.

Perhaps unsurprisingly, the comparable representative-agent economy (Model 5) generates far too much consumption smoothing, which shows up here in the very low values of both the standard deviation of aggregate consumption and its correlation with output. More

[26] We simulated 10 000 households for 11 000 periods and discarded the first 1 000 periods.

interestingly, the hand-to-mouth model (Model 3) also underestimates the volatility of consumption but overestimates its correlation with output, relative to both the data (first row) and the two incomplete-market economies (Models 2 and 4). The first feature was discussed in Section 3.1 above (see equations (24) and (24)): because impatient households hold no precautionary wealth in Model 3, current individual wealth is entirely unresponsive to changes in *future* labour market conditions, while it does under time-varying precautionary savings. Incomplete markets and time-varying precautionary savings are thus key in generating high level of consumption volatility relative to the simpler hand-to-mouth economy, thereby taking the model much closer to the volatility of aggregate consumption that is observed in the data. The second feature of the hand-to-mouth model –its high implied output-consumption correlation– is mechanical: since Model 3 has a large fraction of the household consuming their current income in every period, aggregate consumption closely tracks aggregate income.

Economies							*Statistics (%)*			
							Wealth share of bott. Ω%	Std(C)	Corr(Y,C)	Corr(Y,Y_{-1})
1.	Data						.30	.86	80	84
2.	Precautionary-saving model						.30	.78	75	79
3.	Hand-to-mouth model						.00	.59	96	79
4.	Krusell-Smith model						5.5	.72	82	82
5.	Representative-agent model						irr.	.38	50	80
	Sensitivity									
	Ω	$\frac{\delta^I}{w^{I*}}$	κ	β^I	μ					
2.	.6	.6	1.73	.970	.0		.30	.78	75	79
2a.	.3	.6	1.73	.970	.0		.13	.48	75	80
2b.	.6	2/3	1.73	.970	.0		.09	.79	75	79
2c.	.6	.6	1.00	.970	.0		.39	.97	73	79
2d.	.6	.6	1.73	.975	.0		.50	.72	81	79
2e.	.6	.6	1.73	.970	δ^I		-1.75	.77	75	79

Table 2. Summary statistics.

Note: Models 1, 2, 3 and 5 are simulated according to the estimated joint process for (f_t, s_t, z_t). Model 4 is simulated as in Krusell and Smith (1998, Sec. IV). See text for details.

Sensitivity. The bottom part of Table 2 focuses on our baseline precautionary-saving model (Model 2), and reports the sensitivity of the moments under consideration with respects to changes in the share of impatient household (Ω, Model 2a), the replacement ratio (δ^j/w^{*j}, $j = I, P$, Model 2b), the skill premium parameter (κ, Model 2c), the subjective discount factor for impatient households (β^I, Model 2d), and the borrowing limit (μ, Model 2e)

Models 2a and 2b correspond to economies which are closer to the representative-agent model than our baseline specification, and hence where market incompleteness plays a more limited role in driving the response of consumption to aggregate shocks. In Model 2a, the share of impatient households is reduced; unsurprisingly, the greater proportion of permanent-income consumers in this specification implies a lower aggregate consumption volatility and a lower correlation with output, relative to the baseline model. In Model 2b, the number of impatient households is maintained at its baseline value, but these households are endowed with better insurance opportunities; namely, the steady state gross replacement ratio is raised to the value 2/3 (instead of 0.6), while social contributions still adjust (upwards) to satisfy the balanced-budget condition (10). While this value of the replacement ratio is still consistent with positive asset holdings by impatient households (that is, the economy remains a precautionary-saving one), the implied greater level of direct insurance crowds out self-insurance and hence substantially raises wealth dispertion. Aside from this difference, the second-order moment properties of the model are close to those of Model 2.

In Model 2c, the heterogeneity in labour efficiency across patient and impatient households that was assumed in the baseline specification is removed. Since all households earn the same wage, wealth is slightly more equally distributed. More importantly, this specification overestimate the consumption of the 60% poorest in the population, as those end up consuming 53% total consumption (not reported in Table 2), against 41% in the data (see Table 1). Since the consumption of impatient consumpers responds more to aggregate shocks than that of patient (i.e., permament-income) consumers, the composition effect leads to an overestimation of volatility of consumption. Model 2d is one in which impatient households are more patient than in the baseline specification, and consequently hold a larger fraction of total wealth; the volatility properties of that specification are close to those of the baseline specification –and very close to the Krusell-Smith model. Finally, in Model 2e the borrowing limit –which was set to zero in the baseline specification– is relaxed. Recall in the baseline precautionary-saving model (Model 2) only unemployed, impatient households face a binding borrowing limit; here we impose that these households' can borrow up to the amount of the unemployment benefit δ^I. In this economy, even though the employed do hold a buffer stock of wealth in excess of the borrowing limit, this amount of wealth is negative; consequently, impatient households as a whole end up holding a negative fraction of total wealth. Aside from this cross-sectional difference, the time-series properties under study are almost unchanged relative to Model 2.

5 Concluding remarks

In this paper, we have proposed a tractable general equilibrium model of households' behaviour under incomplete insurance and time-varying precautionary savings, and then gauged its ability to shed light on the dynamics of aggregate consumption over the business cycle. In contrast to earlier attempts at constructing tractable versions of models with heterogenous agents, the specificity of ours has been to combine i. a realistic representation of households' labour income risk –modelled as resulting from the combined effects of persistent changes in the equilibrium real wage and in the probabilities to transit across employment statuses; and ii. the reduction of the model's dynamic to a small-scale system solved under rational expectations, thanks to exact cross-household aggregation. Finally, the model has been calibrated to match the broad features of the cross-sectional wealth and consumption dispertions that are observed in the US economy, and then fed with joint productivity and labour market shocks –such as estimated from post-war data. Despite its simplicity, the model was found to do a fairly reasonable job at explaning the time-series behaviour of aggregate consumption. In particular, the comparision with the pure "hand-to-mouth" model reveals that time-variations in precautionary savings may significantly raise consumption volatility, even though the individual wealth of precautionay savers (as a share of aggregate liquid wealth) is low on average.

While this paper has deliberately focused on a simple model specification (by drastically limiting the number of household types) and quantified its most direct implication (the implied time-series properties of aggregate consumption), the tractability of our framework may be useful other contexts. For example, it can easily be used to interact incomplete markets with other frictions (e.g., nominal rigidities, labour market frictions) that are widely believed to matter for the business cycle. Moreover, it can handle a potentially large number of state variables with continuous support, which is empirically attractive (e.g., for structural estimation).

Appendix

A. Endogenous transitions rates

This appendix shows how (f_t, s_t) can be determined by the job creation policy of the firm in a labour market with search and matching frictions. We use the same timing convention and employment contract as in Hall (2009), which allows us to be as close as possible to our baseline model with exogenous transition rates. Moreover, for expositional conciseness we assume that $\kappa = 1$ (i.e., all households are equally productive), but the argument directly extends to the case where $\kappa \neq 1$.

More specifically, our timing is as follows. At the very beginning of date t, a fraction ρ of existing employment relationships are broken, thereby creating a job seekers' pool of size $1 - (1-\rho) n_{t-1}$ (that is, unemployment at the end of date $t-1$, $1 - n_{t-1}$, *plus* the broken relationships at the beginning of date t, ρn_{t-1}). Members of this pool then have a probability f_t to find a job within the same period, and stay unemployed until the end of the period with complementary probability. It follows that the period-to-period separation rate is $s_t = \rho(1 - f_t)$.

The job-finding rate, f_t, is determined as follows. Given its knowledge of $1 - (1-\rho) n_{t-1}$ and z_t, the representative firm posts v_t vacancies at cost $c > 0$ each and a fraction λ_t of which are filled in the current period. Total employment at the end of date t is thus:

$$n_t = (1-\rho) n_{t-1} + \lambda_t v_t. \tag{37}$$

The vacancy filling rate λ_t is related to the vacancy opening policy of the firm via the matching technology. The number of matches M_t formed at date t is assumed to depend on both the size of the job seekers' pool and the number of posted vacancies, v_t, according to the function $M_t = M(1 - (1-\rho) n_{t-1}, v_t)$, which is increasing and strictly concave in both arguments and has CRS. Thus, the vacancy-filling rate satisfies $\lambda_t = M_t / v_t = m(\theta_t)$, where $\theta_t \equiv v_t / (1 - (1-\rho) n_{t-1})$ is the market tightness ratio, and where the function $m(\theta_t) \equiv M(\theta_t^{-1}, 1)$ is strictly decreasing in θ_t.

Once matched, the households and the firm split the match surplus according to bilaterally efficient dynamic contracts that are negotiated at the time of the match and implemented as planned for the duration of the match. Following Hall (2009) and Stevens (2004), we restrict our attention to a simple class of dynamic contracts whereby the firm pays the worker its full marginal product, except at the time of the match when the worker is paid below marginal product. The profit flow extracted by the firm on new matches motivates –and finances– the payment of vacancy opening costs, but existing matches generate no quasi-rents thereafter. Formally, this arrangement is equivalent to a "fee contract" in which any matched worker i enjoys the wage $w_t = z_t G_2(K_t, n_t)$ at any point in time but pays a fixed fee $\psi_t > 0$ to the firm at the time of hiring; that is, the worker is actually paid $\tilde{w}_t = w_t - \psi_t > 0$ during the (one-period) probation time and the full wage thereafter. We let ψ_t respond to the aggregate state, i.e., $\psi_t = \psi(z_t)$, $\psi'(.) > 0$. The representative firm maximises its instantaneous profit flow

$$\Pi_t = z_t G(K_t, n_t) - n_t w_t - (R_t - 1 + \mu) K_t + v_t (\lambda_t \psi_t - c), \tag{38}$$

subject to (37), and taking n_{t-1}, λ_t, R_t, as well as the contract $(\tilde{w}_t, w_t)$, as given. The optimal choice of capital per employee is given by equation (7). On the other hand, from (38) the firm expands vacancy openings until $z_t G_2(K_t, n_t) \lambda_t - \lambda_t w_t + \lambda_t \psi_t - c = 0$. Since $z_t G_2(K_t, n_t) = w_t$ in the class of contracts under consideration, the economywide vacancy-filling rate that results from these openings is:

$$\lambda_t = c/\psi(z_t) \equiv \lambda(z_t), \ \lambda'(.) < 0. \tag{39}$$

From (38)–(39) and the CRS assumption, the firm makes no pure profits in equilibrium (i.e., old matches generate no profit, while the quasi-rent extracted from new matches is exhausted in the payment of vacancies costs). From the matching function specified above, the tightness ratio that results from the optimal vacancy policy of the firm is $\theta_t = m^{-1}(\lambda_t)$. Hence, the job-finding rate in this economy is:

$$f_t = \lambda_t . m^{-1}(\lambda_t) \equiv f(z_t), \ f'(.) > 0, \tag{40}$$

and the job separation rate is $s(z_t) = \rho(1 - f(z_t))$. Note that under this structure the firm's problem is static and thus unambiguous despite the fact that impatient and patient households do not share the same pricing kernel.

B. Equilibria with many wealth states

In this Appendix, we use the same constructive approach as in the body of the paper in order to derive a set of necessary and sufficient conditions for the existence and uniqueness of equilibria with many wealth states. We derive these conditions at the steady state (which as before satisfies $R^* = 1/\beta^P$, (28) and (33)), knowing that those will still hold in the vicinity of the steady state provided that aggregate shocks are not too large. More specifically, we generalise our approach to construct equilibria having the following properties: i) given a period utility function for impatient households with the shape depicted in Figure 1, i) all unemployed household consume less than c^*; ii) all employed households consume more than c^*; iii) none of the employed face a binding borrowing constraint; and iv) unemployed households do face a binding borrowing constraint after $m < \infty$ consecutive periods of unemployment (that is, asset liquidation takes place gradually rather than in one period, but is achieved in finite time). For expositional clarity we focus on the case where $\mu = 0$ (i.e., the unemployed cannot borrow), but the argument can straightforwardly generated to the case where $\mu > 0$.

Let us call 'k-unemployed households' those who are unemployed for exactly $k \geq 1$ consecutive periods at date t (and thus who were still employed at date $t-k$). For example, 1-unemployed households are unemployed at date t and but were employed at date $t-1$. Similarly, we denote by 'k-employed households' those employed at date t who where unemployed from date $t-k-1$ to date $t-1$. By extension, 0-employed households are those employed date t who were also employed at date $t-1$.

Conjecture that i) all k-unemployed households consume the same amount c_k^u and hold the same asset wealth, denoted a_k; ii) all employment households hold the same asset wealth, denoted a_0; and iii) all k-employed households consume the same amount c_k^e. We will show that it is indeed the case in equilibrium. By assumption, the period utility function has the following property:

$$u(c) = \begin{cases} \tilde{u}(c) & \text{if } c \leq c^* \\ \tilde{u}(c^*) + \eta(c - c^*) & \text{if } c > c^* \end{cases} \tag{41}$$

Moreover we assume that $\tilde{u}$ is differentiable and strictly concave, with $\tilde{u}'(c) > \eta$ for all $c < c^*$ and $\lim_{c \to 0^+} \tilde{u}'(c) = +\infty$. Finally, we assume that:

$$\eta < \beta^I R^* \left[(1 - s^*)\eta + s^* \tilde{u}'\left(\delta^I\right)\right] \tag{42}$$

We will first characterize the equilibrium wealth distribution when (42) holds. Then we will show that when it does not, then all impatient households face a binding borrowing limit and hence simply behave like 'rule-of-thumb' consumers.

Consumption and asset levels. Under our conjectured equilibrium, the Euler equations determining the distribution of wealth are:

$$\eta = \beta^I \left[(1 - s^*)\eta + s^* \tilde{u}'\left(c_1^u\right)\right] R^*, \tag{43}$$

$$\tilde{u}'\left(c_k^u\right) = \beta^I \left[f^* \eta + (1 - f^*) \tilde{u}'\left(c_{k+1}^u\right)\right] R^* \text{ if } m \geq 2 \text{ and for } k = 1...m-1, \tag{44}$$

where (43) is the Euler equation for employed households and (44) are the Euler equations for unemployed households before the borrowing constraint starts binding. From the households' budget constraints, the consumption levels of unemployed households are

$$c_k^u = \delta^I + a_{k-1}R^* - a_k \text{ if } m \geq 2 \text{ and for } k = 1...m-1, \tag{45}$$

$$c_m^u = \delta^I + a_{m-1}R^*, \tag{46}$$

$$c_k^u = \delta^I \qquad\qquad \text{for } k > m, \tag{47}$$

while those of employed households are

$$c_j^e = w^* (1 - \tau^*) + a_j R^* - a_0 \text{ for } j = 0, ...m \tag{48}$$

The conditions for the existence of an equilibrium with $m+1$ wealth states can be stated as follows:

$$c_k^e > c^* > c_j^u \text{ for all } k \geq 0 \text{ and } j \geq 1, \tag{49}$$

$$a_k > 0 \text{ for } k \geq 0, \tag{50}$$

$$\tilde{u}'(c_m^u) > \beta [f^* \eta + (1 - f^*) \tilde{u}'(\delta)] R^*. \tag{51}$$

Inequality (49) ensures that the ranking of consumption levels is consistent with the assumed instant utility function. Inequality (50) states that asset holdings are positive before the constraint binds. Equation (51) states that the borrowing constraint is binding after m periods of unemployment. We now derive a set of necessary and sufficient conditions for equations (43)–(51) to hold, thereby implying the existence of an equilibrium with limited cross-sectional heterogeneity. Moreover, we will show that when this equilibrium exist, then it is unique.

Existence and uniqueness of the equilibrium. To establish existence and uniqueness, we first construct a function $F(m, a_{m-1})$ that depends on the number of liquidation periods for unemployed households before the constraint binds (m) and the last strictly positive wealth level before the constraint binds (a_{m-1}). If the borrowing constraints binds after m consecutive periods of unemployment, then we know that $a_m = 0$. In words, for every possible m, F is defined over all possible values of a_{m-1}. The goal is to use F to show that both m and a_{m-1} are unique given the deep parameters of the model. To construct F, note that from (44) and (45) we have

$$\tilde{u}'\left(c_{m-1}^u\right) = \beta^I \left[f^* \eta + (1 - f^*) \tilde{u}'\left(\delta^I + a_{m-1}R^*\right)\right] R^*.$$

Using (44) $m-2$ times to eliminate c^u_{m-1}, c^u_{m-2} and c^u_2, and then (43) to eliminate c^u_1 , we obtain:

$$\left[\beta^I R^* (1-s^*) - 1\right] \eta + s^* f^* \eta \left(\beta^I R^*\right)^2 \left(\sum_{k=0}^{m-2} \left(\beta^I R^* (1-f^*)\right)^k\right) \tag{52}$$
$$+ \left(\beta^I R^*\right) \left(\beta^I R^* (1-f^*)\right)^{m-1} s^* u' \left(\delta^I + a^u_{m-1} R^*\right) = 0$$

The previous equality is valid for $m \geq 2$. If $m = 1$, one can directly use (43) to find a_0, which corresponds to the baseline case analysed in the body of the paper.

Lemma 1 *Define the function* $F : \mathbb{N}\times] - \delta/R^*; +\infty[\to \mathbb{R}$*, such that*

$$F(1,x) \equiv \beta^I R^* s^* \tilde{u}' \left(\delta^I + xR^*\right) - \left[1 - \beta^I R^* (1-s^*)\right] \eta$$

and, for $m \geq 2$*,*

$$F(m,x) \equiv \frac{s^* f^* \eta \left(\beta^I R^*\right)^2}{1-\beta^I R^* (1-f^*)} - \left[1 - \beta^I R^* (1-s^*)\right] \eta \tag{53}$$
$$+ s^* \beta^I R^* \left[\tilde{u}' (\delta + xR^*) - \frac{f^* \eta \beta^I R^*}{1-\beta^I R^* (1-f^*)}\right] \left(\beta^I R^* (1-f^*)\right)^{m-1}.$$

Then, for any $m \geq 1$*,* a_{m-1} *satisfies* $F(m, a_{m-1}) = 0$*.*

The function F is obtained by factorising the left hand side of (52), and it only depends on the deep parameters of the model. The following lemma uncovers some useful properties of F.

Lemma 2 *If (42) holds then*

i) F *is strictly decreasing in both argument*

ii) For all $m \geq 1$*, there exists a unique* x_{m-1} *such that* $F(m, x_{m-1}) = 0$*. Moreover, the series* x_{m-1} *is strictly decreasing in* m *and we have* $x_0 > 0$ *and* $\lim_{m\to\infty} x_{m-1} = -\delta^I/R^*$*.*

Proof. i) We first note that $\tilde{u}' \left(\delta^I + xR^*\right) - \frac{\beta^I R^* f^* \eta}{(1-\beta^I R^*(1-f^*))} > 0$, since $1 > \beta^I R^*$ and $\tilde{u}'(.) \geq \eta$. Using this inequality, the expression for $F(1,x)$ above and equation (53) with $m = 2$, we have $F(1,x) > F(2,x)$. Moreover, for all $m \geq 2$ and $x > -\delta^I/R^*$ we have $F(m-1,x) > F(m,x)$, that is, $F(m,x)$ is strictly decreasing in m. Finally, $F(m,x)$ is decreasing in x for all m because $\tilde{u}' < 0$. ii) Note first that $\lim_{x\to-\delta^I/R^*} F(m,x) = +\infty$ for all $m \geq 1$. Moreover, (42) implies that $F(1,0) = \beta^I R^* s^* \tilde{u}' \left(\delta^I\right) - \left[1 - \beta^I R^* (1-s^*)\right] \eta < 0$. Since $F(1,x)$ is continuous and monotonic in x, by the theorem of continuous functions there is an x_0 such that $F(1,x_0) = 0$. Now, since $F(m,x_0)$ is decreasing in m we have $F(2,x_0) < 0$; and, since

$F(2,x)$ is continuous and monotonic in x, there exists a unique x_1 such that $F(2,x_1)=0$. Then, by induction it is easy to show that $F(m,x_{m-2})<0$, and that, for all $m\geq 1$, there exists a unique x_{m-1} such that $F(m,x_{m-1})=0$. Since $F(m,x)$ is strictly decreasing in both argument, the series x_{m-1} defined by $F(m,x_{m-1})=0$ is decreasing in m. Finally, using (53), one finds that $F(m,x_{m-1})=0$ is equivalent to

$$\begin{aligned}\tilde{u}'\left(\delta^I+x_{m-1}R^*\right)=&\frac{1}{s^*\beta R^*}\left(\left[1-\beta R^*\left(1-s^*\right)\right]\eta-\frac{s^*f^*\eta\left(\beta R^*\right)^2}{1-\beta R^*\left(1-f^*\right)}\right)\frac{1}{\left(\beta R^*\left(1-f^*\right)\right)^{m-1}}\\&+\frac{f^*\eta\beta R^*}{1-\beta R^*\left(1-f^*\right)}\end{aligned}\tag{54}$$

The right hand side of this equation goes to $+\infty$ as m goes to $+\infty$. Since $\lim_{c\to 0^+}\tilde{u}'(c)=+\infty$, this implies that $x_{m-1}\to-\delta^I/R^*$ as $m\to+\infty$. ■

Using the properties of F, we may prove the uniqueness of the equilibrium, provided that the latter exists.

Proposition 2. *If there exists an equilibrium with limited heterogeneity, then it is unique. Moreover, borrowing constraints bind after a finite number of consecutive periods of unemployment.*

Proof. Define

$$m^*=\max\{m|x_{m-1}\geq 0\},\tag{55}$$

that is, m^* is the largest m for which x_{m-1} is nonnegative. Since x_{m-1} is strictly decreasing, $x_0>0$ and $\lim_{m\to+\infty}x_{m-1}=-\delta^I/R^*$, m^* is finite and uniquely defined. We now show (by contradiction) that if there exists an equilibrium with limited heterogeneity, then the borrowing constraint must be binding after exactly m consecutive periods of unemployment. Suppose that there is also an equilibrium in which households face binding borrowing constraints after $n\neq m^*$ consecutive periods of unemployment. Then n would satisfy $F(n,a_{n-1})=0$. First, it is impossible that $n>m^*$, since in this case we would have $a_{n-1}<0$, a contradiction. Second, if we could find $n<m^*$ then we would have $F(m^*,a_{m^*-1})=F(n,a_{n-1})\,(=0)$. Rearranging this equality, it can be shown that it would imply that if the borrowing constraint binds after n consecutive periods of unemployment then $a_{m-1}<0$, a contradiction again. Thus, the only possible equilibrium is such that $m=m^*$, where m^* is finite. ■

We now characterize the equilibrium allocation with m^* liquidation periods.

Lemma 3 *In an equilibrium with limited heterogeneity, i) c_k^u is strictly decreasing in $k=1...m^*$; and ii) a_j is strictly decreasing in $j=0...m^*-1$*

Proof. i) is proven by induction. From (44), if $\tilde{u}'(c_k^u) > \eta f / \left(1/\beta^I R^* - 1 + f\right)$, then $\tilde{u}'\left(c_{k+1}^u\right) > \tilde{u}'(c_k^u)$ and hence $c_{k+1}^u < c_k^u$. Now, from (43) we have

$$\tilde{u}'(c_1^u) = \left(1 + \frac{1/(\beta R^*) - 1}{s^*}\right)\eta.$$

Since $\beta^I R^* < 1$, we know that $\tilde{u}'(c_1) > \eta$; and since $f/\left(1/\beta^I R^* - 1 + f\right) < 1$, it implies that $\tilde{u}'(c_1) > \eta f/\left(1/\beta^I R^* - 1 + f\right)$. ii) For $m^* = 1$, then from (42) we have $a_0 > 0$. For $m^* = 2$, since $c_{m^*-1}^u = \delta^I + a_{m^*-2}R^* - a_{m^*-1} > c_m^u = \delta^I + a_{m^*-1}R^*$ we have $a_{m^*-2} > a_{m^*-1}(1 + 1/R^*) > a_{m^*-1}$. For $m \geq 3$, we again reason by induction. First, $a_{m^*-2} > a_{m^*-1}$, for the same reason as when $m^* = 2$. Second, if $a_{m^*-2} > a_{m^*-1}$ then $a_{m^*-3} > a_{m^*-2}$; Indeed, from (45)-(47) and the fact that $c_{k-1}^u > c_k^u$, $k = 0, ...m^* - 1$ we have $a_{m^*-3} - a_{m^*-2} > (a_{m^*-2} - a_{m^*-1})/R^*$. ∎

We may then construct the states of the wealth distribution as follows. Given m^* in (55), the sequence $\{a_{m^*-i}\}_{i=0}^{m^*}$ is given by $a_{m^*} = 0$, a_{m^*-1} that solves $F(m^*, a_{m^*-1}) = 0$, and then the recursion

$$\begin{aligned} &\tilde{u}'\left(\delta^I + R^* a_{k-2} - a_{k-1}\right) \\ &= \beta^I \left[f^*\eta + (1 - f^*)\,\tilde{u}'\left(\delta^I + R^* a_{k-1} - a_k\right)\right] R^* \text{ if } m^* \geq 2 \text{ and for } 2 \leq k \leq m^* \end{aligned} \tag{56}$$

until a_0. This recursion also allows us to find a_0 as a function of the deep parameters of the model. Indeed, define the function G and net dis-saving X_k as follows:

$$G(X) = \tilde{u}'^{-1}\left(\beta^I R^* \left[f^*\eta + (1 - f^*)\,\tilde{u}'\left(\delta^I + X\right)\right]\right) - \delta^I, \tag{57}$$

$$X_k \equiv R^* a_{k-1} - a_k. \tag{58}$$

Then, (56) can be written as $X_{k-1} = G(X_k)$ for $k = 2..m^*$. Iterating this equation gives:

$$X_k = G^{(m^*-k)}(R^* a_{m^*-1}) \text{ for } k = 1...m^* - 1,$$

where $G^{(i)}$ is the i-th iteration of G. The a_ks are then recovered from (58). In particular,

$$a_0 = \sum_{j=1}^{m^*-1} \frac{G^{(j)}(R^* a_{m^*-1})}{(R^*)^j} + \frac{a_{m^*-1}}{(R^*)^{m^*-1}},\ a_1 = \sum_{j=1}^{m^*-2} \frac{G^{(j)}(R^* a_{m^*-1})}{(R^*)^j} + \frac{a_{m^*-1}}{(R^*)^{m^*-2}} \tag{59}$$

We can now prove the following proposition.

Proposition 3. *If $w^*(1 - \tau^*) - a_0 > c^* > \delta + R^* a_0 - a_1$, where a_0 and a_1 are given (59). Then, the equilibrium with reduced heterogeneity exists and is unique.*

Proof. We have shown that if (42) holds, then there is a unique m^* and a unique sequence c_k^u, $k = 1...m$ such that the Euler equations (43)–(44) hold and the borrowing constraint is binding after m continuous periods of unemployment. For this allocation to be an equilibrium with limited heterogeneity, one has to show that the ranking of consumption levels (49) is satisfied. Since both c_k^u and a_k are decreasing in k, c_j^e in (48) is decreasing in j. Hence, a sufficient condition for the postulated ranking of consumption levels to hold is $c_m^e > c^* > c_1^u$, which is equivalent to $w^* (1 - \tau^*) - a_0 > c^* > \delta + R^* a_0 - a_1$. If this last condition hold, then the allocation $\{c_k^u, a_k\}_k$ is the unique equilibrium. ■

Construction of the period utility function. Taking a utility function $\tilde{u}$ and a coefficient η, we can now state the conditions under which one can construct an equilibrium with limited cross-sectional heterogeneity.

Proposition 4. *For a given function $\tilde{u}$, differentiable, increasing and strictly concave and with $\lim_{c \to 0^+} \tilde{u}'(c) = +\infty$ and a given $\eta > 0$, one can construct an increasing and concave utility function u such that a limited heterogeneity equilibrium exists provided that*
i) $w^* (1 - \tau^*) - a_0 > \delta + R^* a_0 - a_1$,
ii) $\tilde{u}' (\delta + R^* a_0 - a_1) \geq \eta$,
where a_0 and a_1 are given by (59).

Indeed, if i) holds, then one can finds a c^* such that $w^* (1 - \tau^*) - a_0 > c^* > \delta + R^* a_0 - a_1$. One may then construct a utility function u differentiable all over $[0, \infty)$ by smooth-pasting $\tilde{u}(.)$ with the linear part of $u(.)$ in an arbitrarily small neighborhood of c^*. The function u can be (weakly) concave provided that ii) holds.

C. Data

In Section 4, we use quarterly time series for real GDP and consumption, as well labour market transition rates and total factor productivity over the period 1948Q1-2011Q1.

Real GDP is from the NIPA tables, while the real aggregate consumption series is obtained by dividing nominal spending on nondurable goods and services by the PCE price index (all from the NIPA tables). To compute the relevant statistics (row "data" in Table 2), the cycle component of both series are extracted by HP-filtering the log of the series (with smoothing parameter 1600).

For the labour-market transition probabilities, we proceed as follows. First, we compute monthly job-finding probabilities using CPS data on unemployment and short-run unemployment, using the two-state approach of Shimer (2005, 2012). (As suggested by Shimer, 2012, the short-run unemployment series is made homogenous over the entire sample by multiplying the raw series by 1.1 from 1994M1 onwards). We then compute monthly separation probabilities as residuals from a monthly flow equation similar to (1). Using these two series, we construct transition matrices across employment statuses for every month in the sample, and then multiply those matrices over the three consecutive months of each quarter to obtain quarterly transition probabilities (this naturally implies that cyclical fluctuations in the quarter-to-quarter separation rate partly reflect changes in the underlying monthly job-finding rate probability).

Finally, our series for total factor productivity is computed as in Ríos-Rull and Santaeulàlia-Llopis (2010). When estimating the joint behaviour of TFP and labour market transition probabilities, all three series are HP-filtered with penalty parameter 1600.

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